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AERO-SERVO-ELASTIC STABILITY ANALYSIS

by
William P. Rodden, Mildred R. Zeifman
and
John M. Powers, Jr.

Prepared for
DEPARTMENT OF THE NAVY
NAVAL AIR SYSTEMS COMMAND
Washington, D.C. 20361

Under
Contract N00019-76-C-0346
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A general matrix formulation of the aero-servo-elastic stability problem is presented based on the British method of flutter analysis. The analysis applies to the closed-loop system, and the digital computer program (MPASES) developed to perform the analysis is described. An application is made to a typical air-to-air missile with three feedback loops.

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SUMMARY

A general matrix formulation of the aero-servo-elastic stability problem for a closed-loop system is presented. It is based on a classical method for determining servomechanism stability (using linear differential equations) and the British method of flutter analysis which represents the aerodynamic forces as frequency dependent springs and dampers. The combination of the aerodynamic forces with the mechanical springs and dampers in the equations of motion and then with their electro-mechanical equivalents in the servo system leads to a consistent formulation of the linear equations of motion for the closed-loop aero-servo-elastic system. An iterative real eigenvalue solution accounts for the frequency dependence of the aerodynamic forces which is only secondary.

The digital computer program MPASES (modified program for aero-servo-elastic stability) developed to perform the analysis is described. Dynamic storage allocation is utilized throughout MPASES to provide the most efficient use of core storage through the variable dimensioning of all arrays. Program limitations are minimal allowing stability results in one computer run to be obtained for up to ten velocities at each of five altitudes. In addition, as many as twenty-five servo element gain variations are permitted.

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SECTION I

INTRODUCTION

This report documents a modification to Ref. 1 which presented a digital computer program for the analysis of aero-servo-elastic system stability. Reference 1 formulated the problem by combining one of the classical methods for determining servomechanism system stability with the American method of flutter analysis. Since the American method of flutter analysis represents the aerodynamic forces as frequency dependent masses and utilizes the mathematical concept of an artificial structural damping to determine the reduced frequency at flutter, the combined aero-servo-elastic stability analysis does not obtain physically meaningful frequencies and dampings at flight conditions other than neutrally stable ones. On the other hand, the British method of flutter analysis represents the aerodynamic forces more realistically as frequency dependent springs and dampers. This is only an approximation for transient motion since the aerodynamic forces are generally known only for harmonic motion and it is not completely correct to identify the forces in phase with the displacements as aerodynamic springs and the forces in phase with the velocities as aerodynamic dampers. However, comparisons of results from the various methods for flutter analysis have been made by Jocelyn Lawrence and Jackson in Ref. 2, and it has been found that the British method of flutter analysis leads to reasonable predictions of transient aeroelastic behavior when the damping levels are low.

The British method of treating the aerodynamic loads as springs and dampers permits combining the aerodynamic forces with the mechanical springs and dampers in the equations of motion and then with their electro-mechanical equivalents in the servo system. This results in a consistent formulation of the equations of motion for the closed-loop aero-servo-elastic system because the frequency dependence of the aerodynamic forces is only secondary.

Although the frequency dependence is secondary, however, it is important and an iteration is necessary to "line-up" the reduced frequency for which the aerodynamics are determined and the frequency determined by the equations of motion; one iteration, however, is sufficient to achieve sufficient accuracy between the aerodynamic forces and the equations of motion.

The iteration begins at a particular velocity by assuming the reduced frequency k=0. (The aerodynamic damping can only be found from a low frequency k≈0.0 but an interpolation scheme will estimate the damping at k=0.) Solving the eigenvalue problem from the equations of motion yields either separate and real roots or complex conjugate pairs. Any real roots found for k=0 are correct, e.g., the rigid body roll-damping root or a structural divergence root. However, the complex conjugate roots must be lined up and the lowest frequency roots permit the aerodynamic forces to be revised (by interpolation) to the corresponding reduced frequency. With the revised aerodynamics, the eigenvalue problem is solved again and the lowest oscillatory root is obtained along with an estimate of the next higher frequency. The aerodynamic forces are then revised again to correspond to the estimate of the reduced frequency of the second oscillatory mode and the new eigenvalue problem is solved. This results in the second oscillatory root and an estimate of the third frequency. The process of revising the aerodynamics and solving new eigenvalue problems continues until the desired number of roots have been obtained for the particular velocity. The whole procedure is repeated for the next higher velocity and is continued until the velocity range of interest has been covered. From the solutions for frequency and damping of each mode, root locus charts can be drawn for use in redesign of servo components, e.g., amplifier gains, or mechanical components, e.g., mass balancing.

The modified program for aero-servo-elastic stability (MPASES) requires four revisions to the program PASES of Ref. 1. The first is the inclusion of the aerodynamic forces as spring and damping terms in the equations of motion rather than as complex inertial terms. The second is the iterative eigenvalue solution required to line up the frequencies between the aerodynamic forces and the equations of motion as discussed above. The third problem is the interpolation of the aerodynamic forces necessary in the iteration to minimize aerodynamic computational expense. The last problem is the result of the new formulation dealing only with real matrices; a specialized eigenvalue extraction method that analyzes real matrices which have either real or complex conjugate roots may be utilized for computational efficiency. Each of these modifications is discussed in the following sections.

SECTION II

CLOSED-LOOP AERO-SERVO-ELASTIC STABILITY ANALYSIS

New Equations of Motion

The theoretical derivation of the equations of motion for the PASES computer program is taken from Ref. 1 and is reproduced in this report in the Appendix for ease of reference. It is only necessary here to rederive the aeroelastic equations of motion since the servo equations are not affected by the change in representation of the aerodynamic forces from complex masses to real springs and dampers.

Our new definitions of aerodynamic influence coefficients (AIC's) are taken from the survey of unsteady AIC's in Ref. 3. The unsteady force is given in terms of the deflections and their velocities by

$$\{F\} = (qS/\bar{c})\left([C_{hs}]\{h\} + [C_{hDh}]\{\dot{h}\bar{c}/V\}\right) \tag{1}$$

The AIC's $[C_{hs}]$ and $[C_{hDh}]$ may be regarded as static and dynamic stability derivatives but are, in general, functions of the reduced frequency

$$k = \omega b_r/V \tag{2}$$

where

$$b_r = \bar{c}/2 \tag{3}$$

Assuming the AIC's to be constants independent of k simplifies the analysis and is adequate for low frequencies, but it is inaccurate at higher frequencies and the reduced frequencies should be lined up with the frequencies obtained from the equations of motion as discussed in the Introduction. However, the choice of lining up the frequencies or not is left to the user in his choice of frequency dependence of the AICs.

A definition of complex oscillatory AIC's was also given in Ref. 3 as

$$\{F\} = \rho\omega^2 b_r^2 s [C_h]\{h\} \tag{4}$$

for use in the American method of flutter analysis. Equations (1) and (4) must be identical for harmonic motion and we have

$$(qS/\bar{c})\Big([C_{hs}] + i(\omega\bar{c}/V)[C_{hDh}]\Big)\{h\} = \rho\omega^2 b_r^2 s\Big([C_{hR}] + i[C_{hI}]\Big)\{h\} \tag{5}$$

where $[C_{hR}]$ and $[C_{hI}]$ are the real and imaginary parts of $[C_h]$, respectively. Identifying the real and imaginary parts of Eq. (5) leads to

$$[C_{hs}] = 2k^2(\bar{c}s/S)[C_{hR}] \tag{6}$$

and

$$[C_{hDh}] = k(\bar{c}s/S)[C_{hI}] \tag{7}$$

Equations (6) and (7) determine the aerodynamic stiffness and damping AIC's when the oscillatory AIC's are given in the format of Eq. (4), as is the case with a number of available computer programs, e.g., Refs. 4 and 5.

In terms of the stiffness and damping AIC's, the equations of motion of the aeroelastic system including the rotations of control surfaces appear as (cf. Appendix A, Eq. (2-15))

$$[M]\{\ddot{h}\} + [C]\{\dot{h}\} + [K](\{h\} - [h_\delta]\{\delta\}) = \{F\} \tag{8a}$$

$$= (qS/\bar{c})\Big([C_{hs}]\{h\} + [C_{hDh}]\{\dot{h}\bar{c}/V\}\Big) \tag{8b}$$

or

$$[M]\{\ddot{h}\} + [\bar{C}]\{\dot{h}\} + [\bar{K}]\{h\} - [K][h_\delta]\{\delta\} = 0 \tag{9}$$

where

$$[\tilde{C}] = [C] - \tfrac{1}{2}\rho VS[C_{hDh}] \tag{10}$$

$$[\tilde{K}] = [K] - (qS/\bar{c})[C_{hs}] \tag{11}$$

The modal solution proceeds as before (see Appendix A). The series for the deflections is

$$\{h\} = [h_F]\{a_F\} + [h_R]\{a_R\} + [h_\delta]\{\delta\} \tag{12}$$

Substituting Eq. (12) into Eq. (9) and premultiplying by $[h_F]^T$ leads to the modal equations for the flexible degrees of freedom.

$$\begin{aligned} &[\mathcal{M}_F]\{\ddot{a}_F\} + [\mathcal{M}_{FR}]\{\ddot{a}_R\} + [\mathcal{M}_{F\delta}]\{\ddot{\delta}\} \\ &+ [\tilde{\mathcal{C}}_F]\{\dot{a}_F\} + [\tilde{\mathcal{C}}_{FR}]\{\dot{a}_R\} + [\tilde{\mathcal{C}}_{F\delta}]\{\dot{\delta}\} \\ &+ [\tilde{\mathcal{K}}_F]\{a_F\} + [\tilde{\mathcal{K}}_{FR}]\{a_R\} + [\tilde{\mathcal{K}}_{F\delta}]\{\delta\} = 0 \end{aligned} \tag{13}$$

where

$$[\mathcal{M}_F] = [h_F]^T[M][h_F] \tag{14}$$

$$[\mathcal{M}_{FR}] = [h_F]^T[M][h_R] \tag{15}$$

$$[\mathcal{M}_{F\delta}] = [h_F]^T[M][h_\delta] \tag{16}$$

$$[\tilde{\mathcal{C}}_F] = [h_F]^T[\tilde{C}][h_F] \tag{17}$$

$$[\tilde{\mathcal{C}}_{FR}] = [h_F]^T[\tilde{C}][h_R] \tag{18a}$$

$$= -\tfrac{1}{2}\rho VS[h_F]^T[C_{hDh}][h_R] \tag{18b}$$

$$[\bar{\mathcal{C}}_{F\delta}] = [h_F]^T[\bar{C}][h_\delta] \tag{19a}$$

$$= -\tfrac{1}{2}\rho VS[h_F]^T[C_{hDh}][h_\delta] \tag{19b}$$

$$[\bar{\mathcal{K}}_F] = [h_F]^T[\bar{K}][h_F] \tag{20}$$

$$[\bar{\mathcal{K}}_{FR}] = [h_F]^T[\bar{K}][h_R] \tag{21a}$$

$$= -(qS/\bar{c})[h_F]^T[C_{hs}][h_R] \tag{21b}$$

$$[\bar{\mathcal{K}}_{F\delta}] = [h_F]^T[\bar{K}][h_\delta] \tag{22a}$$

$$= -(qS/\bar{c})[h_F]^T[C_{hs}][h_\delta] \tag{22b}$$

Equation (13) may be compared to Eq. (2-20) in Appendix A. We have utilized the fact above that the rigid body displacements cause no internal damping or structural forces. We have assumed the vibration modes are either free-free modes for the entire system or restrained modes for individual components, but may not be arbitrarily chosen modes. The limited orthogonality of the free-free or restrained modes (i.e., $[\mathcal{M}_F]$ is not a diagonal matrix unless all modes are free-free modes) leads to a diagonal form for the generalized stiffness matrix $\lceil \mathcal{K}_F \rfloor$. If we denote the diagonal elements of $[\mathcal{M}_F]$ by $\lceil \mathcal{M}_F \rfloor$, then the generalized stiffness matrix is

$$\lceil \mathcal{K}_F \rfloor = [h_F]^T[K][h_F] \tag{23a}$$

$$= \lceil \omega_F^2 \rfloor \lceil \mathcal{M}_F \rfloor \tag{23b}$$

and

$$[\bar{\mathcal{K}}_F] = \lceil \mathcal{K}_F \rfloor - (qS/\bar{c})[h_F]^T[C_{hs}][h_F] \tag{24}$$

The generalized structural damping matrix does not have a diagonal form, as does the generalized stiffness, but is assumed so as an approximation that is justified by low levels of structural damping. The approximate form of the equivalent viscous structural damping is

$$\lceil C_F \rfloor = \lceil g_F/\omega_F \rfloor \lceil K_F \rfloor \tag{25a}$$

$$= \lceil g_F \rfloor \lceil \omega_F \rfloor \lceil M_F \rfloor \tag{25b}$$

so that

$$[\bar{C}_F] = \lceil C_F \rfloor - \tfrac{1}{4}\rho VS[h_F]^T[C_{hDh}][h_F] \tag{26}$$

Next, substituting Eq. (12) into Eq. (9) and premultiplying by $[h_R]^T$ leads to the modal equations for the rigid body degrees of freedom.

$$\begin{aligned} &[M_{RF}]\{\ddot{a}_F\} + [M_R]\{\ddot{a}_R\} + [M_{R\delta}]\{\ddot{\delta}\} \\ &+ [\bar{C}_{RF}]\{\dot{a}_F\} + [\bar{C}_R]\{\dot{a}_R\} + [\bar{C}_{R\delta}]\{\dot{\delta}\} \\ &+ [\bar{K}_{RF}]\{a_F\} + [\bar{K}_R]\{a_R\} + [\bar{K}_{R\delta}]\{\delta\} = 0 \end{aligned} \tag{27}$$

where

$$[M_{RF}] = [M_{FR}]^T \tag{28}$$

$$[M_R] = [h_R]^T[M][h_R] \tag{29}$$

$$[M_{R\delta}] = [h_R]^T[M][h_\delta] \tag{30}$$

$$[\bar{C}_{RF}] = [h_R]^T[\bar{C}][h_F] \tag{31a}$$

$$= -\tfrac{1}{4}\rho VS[h_R]^T[C_{hDh}][h_F] \tag{31b}$$

$$[\bar{C}_R] = [h_R]^T[\bar{C}][h_R] \tag{32a}$$

$$= -\tfrac{1}{2}\rho VS[h_R]^T[C_{hDh}][h_R] \tag{32b}$$

$$[\bar{C}_{R\delta}] = [h_R]^T[\bar{C}][h_\delta] \tag{33a}$$

$$= -\tfrac{1}{2}\rho VS[h_R]^T[C_{hDh}][h_\delta] \tag{33b}$$

$$[\bar{K}_{RF}] = [h_R]^T[\bar{K}][h_F] \tag{34a}$$

$$= -(qS/\bar{c})[h_R]^T[C_{hs}][h_F] \tag{34b}$$

$$[\bar{K}_R] = [h_R]^T[\bar{K}][h_R] \tag{35a}$$

$$= -(qS/\bar{c})[h_R]^T[C_{hs}][h_R] \tag{35b}$$

$$[\bar{K}_{R\delta}] = [h_R]^T[C_{hs}][h_\delta] \tag{36a}$$

$$= -(qS/\bar{c})[h_R]^T[C_{hs}][h_\delta] \tag{36b}$$

and we have again noted that rigid body displacements produce no internal damping or structural forces. Equation (27) should be compared to Eq. (2-29) in Appendix A.

The new matrix partitions in the aero-servo-elastic equations of motion now become

$$[X_{\ddot{x}}]\{\ddot{x}\} = \begin{bmatrix} [M_F] & [M_{FR}] & [M_{F\delta}] & 0 \\ [M_{RF}] & [M_R] & [M_{R\delta}] & 0 \\ 0 & 0 & \lceil CC \rfloor & [CS2] \\ \lfloor FSA \rfloor & \lfloor RSA \rfloor & [SC] & [SS2] \end{bmatrix} \begin{Bmatrix} \ddot{a}_F \\ \ddot{a}_R \\ \ddot{\delta} \\ \ddot{e}2 \end{Bmatrix} \tag{37}$$

$$[X_{\dot{x}}]\{\dot{x}\} = \begin{bmatrix} [\bar{\mathcal{C}}_F] & [\bar{\mathcal{C}}_{FR}] & [\bar{\mathcal{C}}_{F\delta}] & 0 \\ [\bar{\mathcal{C}}_{RF}] & [\bar{\mathcal{C}}_R] & [\bar{\mathcal{C}}_{R\delta}] & 0 \\ 0 & 0 & \lceil CC \rfloor & [CS2] \\ \lfloor FSG \rfloor & \lfloor RSG \rfloor & [SC] & [SS2] \end{bmatrix} \begin{Bmatrix} \dot{a}_F \\ \dot{a}_R \\ \dot{\delta} \\ \dot{e}2 \end{Bmatrix} \qquad (38)$$

$$[X_x]\{x\} = \begin{bmatrix} [\bar{\mathcal{K}}_F] & [\bar{\mathcal{K}}_{FR}] & [\bar{\mathcal{K}}_{F\delta}] & 0 \\ [\bar{\mathcal{K}}_{RF}] & [\bar{\mathcal{K}}_R] & [\bar{\mathcal{K}}_{R\delta}] & 0 \\ 0 & 0 & \lceil CC \rfloor & [CS2] \\ 0 & 0 & [SC] & [SS2] \end{bmatrix} \begin{Bmatrix} a_F \\ a_R \\ \delta \\ e2 \end{Bmatrix} \qquad (39)$$

These may be compared with Eqs. (2-43), (2-44) and (2-45) in Appendix A. The coefficient matrices $[X_{\dot{x}}]$ and $[X_x]$ each have five new nonzero partitions as expected from moving the aerodynamic terms from the mass matrix to the stiffness and damping matrices.

The Eigenvalue Problem

The new representation of the aerodynamic forces changes the eigenvalue problem only to the extent that the matrices are real now rather than complex. The equation to be solved is still

$$(\gamma[A] + [B])\{V\} = 0 \qquad (40)$$

where the amplitudes of motion, $\{V\}$, are defined by

$$\{v\} = \{V\}\exp(\gamma t) \qquad (41)$$

and instability occurs when the airspeed and/or control system gains are such that the real part of γ is positive.

Although [A] and [B] are now real matrices, they still may be singular and obtaining the canonical form of the eigenvalue problem by a shift in eigenvalues is still appropriate. We let

$$\gamma = \gamma_o - 1/\lambda \tag{42}$$

where γ_o is an arbitrarily chosen real number, and then the new eigenvalue is

$$\lambda = 1/(\gamma_o - \gamma) \tag{43}$$

and the new eigenvalue problem is

$$\lambda\{V\} = (\gamma_o[A] + [B])^{-1}[A]\{V\} \tag{44}$$

The shift value γ_o is arbitrary to the extent that it must be chosen so the linear combination $\gamma_o[A] + [B]$ is nonsingular. A value which scales [A] to be the same order of magnitude as [B] and of the same sign is recommended.

The eigenvalues of Eq. (44) are either real or complex conjugates. A subroutine ALLMAT (Ref. 6) for complex matrices was used in Ref. 1.

A more recent development for the real case of Eq. (44) is the subroutine EIGRF given in the International Mathematical and Statistical Library (IMSL, Ref. 7). Subroutine EIGRF calls IMSL routine EBALAF to balance the matrix. Then IMSL routine EHESSF reduces the balanced matrix to an upper Hessenberg form and routine EQRH3F computes all of the real and/or complex conjugate pairs of eigenvalues of the Hessenberg matrix.

The IMSL Package is universally used and is usually incorporated into the scientific libraries of major computer systems. When requested, the eigenvectors are found in two iterations by the Inverse Power Method with Shifts (Ref. 8, pp. 323, 626-628) in subroutine EGNVCT (Ref. 9). Subroutine

EGNVCT finds the eigenvecter {u} from a complex matrix [U] and its complex eigenvalue Λ by solving the equation

$$[U - \Lambda I]\{u\} = 0 \tag{45}$$

The subroutine is used in the present development by setting

$$\Lambda = 0 \tag{46}$$

and

$$[U] = \gamma[A] + [B] \tag{47}$$

in Eq. (45). If eigenvectors are requested, all real eigenvalues are used in Eq. (47), but only the complex conjugate eigenvalues with positive imaginary parts (positive frequencies) are used, since the eigenvectors are also complex conjugate pairs.

Writing the complex eigenvalue as

$$\gamma = \mu + i\omega \tag{48}$$

where μ is the decay rate and ω is the damping frequency, we find the cyclic frequency to be

$$f = \omega/2\pi \tag{49}$$

and the fraction of critical damping ζ to be

$$\zeta = -\mu/\sqrt{\mu^2 + \omega^2} \tag{50}$$

For comparison to structural damping levels, twice the damping ratio ζ is a preferable output quantity since

$$\zeta = g/2 \tag{51}$$

for a structurally-damped single degree of freedom oscillator. For a non-oscillatory root a different definition of damping ratio is necessary and we chose the time to half amplitude

$$T_{\frac{1}{2}} = \ell n 2/(-\mu) \tag{52}$$

If the motion is unstable, i.e., $\mu > 0$, Eq. (52) gives the (negative) time to double amplitude. The stability of the oscillatory roots can also be compared using Eq. (52) and this will be an additional output quantity.

Lining Up the Reduced Frequency

The need for lining up the reduced frequency for a specified mode of motion with the frequency determined by the eigenvalue problem for that mode was discussed briefly in the Introduction and at some length in Ref. 2. The necessary equations for the iteration are given in this section.

It is possible to begin the iteration with any value of k. However, a finite value of k may be representative of an oscillatory mode but we are equally interested in static modes. Therefore, we begin with k=0 and any real roots will be determined, e.g., the roll-damping root, a static structural divergence root, and any over-damped roots from control system components. The first oscillatory root for a free vehicle may be either the short period mode or the Dutch roll mode and the choice of k=0 provides a good estimate of that.

Let the complex conjugate pairs of roots be denoted by

$$\gamma_{rs} = \mu_{rs} \pm i\omega_{rs} \tag{53}$$

where r denotes the oscillatory mode number ordered by frequency, ($\omega_{1s} < \omega_{2s} < \ldots$), and s denotes the number of the mode under investigation.

The reduced frequency is

$$k = \omega\bar{c}/2V \tag{54}$$

and the aerodynamic forces are determined for k_s which should be

$$k_s = \omega_{ss}\bar{c}/2V \tag{55}$$

However, ω_{ss} is not known at the outset. From the initial solution with k_0=0, we estimate the nonzero frequencies (ω_{10}, ω_{20}, ω_{30}, ...). Refining the aerodynamics by interpolation for $k_1 = \omega_{10}\bar{c}/2V$ and repeating the eigenvalue solution, we find a new set of frequencies (ω_{11}, ω_{21}, ω_{31}, ...). The value of ω_{11} is taken as the correct value for the first mode and its damping α_{11} determines its stability. The eigenvectors for the first root ω_{11} may then be calculated if the mode shapes are desired.

The aerodynamics are next determined by interpolation for $k_2 = \omega_{21}\bar{c}/2V$ and the next eigenvalue solution yields the frequencies (ω_{12}, ω_{22}, ω_{32}, ...). The frequency ω_{22} is now taken to be correct and its damping α_{22} measures the second oscillatory mode stability. We continue with $k_3 = \omega_{32}b_r/V$, finding the aerodynamics for k_3, and then the eigenvalues; ω_{32} and α_{32} are assumed to be correct. The process continues until all roots of interest, and all eigenvectors, if requested, have been found.

Care must be exercised in tracking the oscillatory roots because, as the reduced frequency is increased the roots which were real at k=0 may become complex at higher values of k or roots which were complex at k=0 may become real at higher k's. A suitable algorithm is to choose ω_{ss} as the closest value to $\omega_{s,s-1}$, and then to choose $\omega_{s+1,s}$ as the closest <u>higher</u> value to ω_{ss}.

Linear Spline Interpolation of Aerodynamic Terms

In the modal matrix equations of motion, twelve of the partitions depend on the aerodynamic influence coefficients (AIC's) which are functions of the reduced frequency k. As the reduced frequency changes when the roots are tracked in the frequency lining up process, the AIC's will change and interpolation on k is necessary. It is computationally simpler to interpolate the partitions rather than the AIC's. Two of the partitions depend on structural parameters also but these are independent of k and can be carried along in the interpolation. A linear spline is chosen for the interpolation because it offers cubic accuracy and continuity throughout the regions of interpolation and extrapolation.

A linear spline is a mathematical device for interpolating for a function y(x) for all points x, when y is known for a discrete set of points, $y_i = y(x_i)$. The spline passes through all of the known points. The mathematical spline takes its name from the plastic spline used by draftsmen for drawing curves through specified points. If the plastic spline may be regarded as a uniform beam, the linear spline representing it mathematically is a solution to the uniform beam differential equation. The following derivation for the linear spline is taken from Ref. 10, App. E, by R. L. Harder.

We wish to determine the deflection curve of a continuous beam over multiple supports. Consider the fundamental solution to the deflection equation

$$EI\frac{d^4y}{dx^4} = w \tag{S6a}$$

$$= 0 \tag{S6b}$$

that is symmetrical about a support at the origin $x = 0$. The solution is

$$y = A = B|x| + Cx^2 + D|x|^3 \tag{57}$$

Continuity of slope requires $B = 0$, and equilibrium with the support load P requires

$$\lim_{\varepsilon \to 0} \int_{-\varepsilon/2}^{\varepsilon/2} w\,dx = \lim_{\varepsilon \to 0} EI \int_{-\varepsilon/2}^{\varepsilon/2} \frac{d^4 y}{dx^4}\,dx \tag{58a}$$

$$= \lim_{\varepsilon \to 0} EI \left. \frac{d^3 y}{dx^3} \right|_{-\varepsilon/2}^{+\varepsilon/2} \tag{58b}$$

$$= P \tag{58c}$$

From Eq. (57) the third derivative is found to be

$$\frac{d^3 y}{dx^3} = 6D \operatorname{sgn} x \tag{59}$$

and Eq. (58b) becomes

$$(12EI)\ D = P \tag{60}$$

Combining Eqs. (57) and (60) and generalizing for all supports at $x = x_i$ leads to

$$y(x) = \sum \left(A_i + C_i (x - x_i)^2 + Q_i |x - x_i|^3 \right) \tag{61}$$

where $Q_i = P_i/12EI$. At a large distance from the supports, the deflection curve should be linear. For large $x \to \pm\infty$, Eq. (61) behaves like

$$y(x) = x^3 \operatorname{sgn} x \sum Q_i + x^2 \sum (C_i - 3Q_i x_i \operatorname{sgn} x) + O(x) \tag{62}$$

and the required linear behavior necessitates

$$\sum Q_i = 0 \tag{63}$$

$$\sum C_i = 0 \tag{64}$$

$$\sum Q_i x_i = 0 \ . \tag{65}$$

Equations (63) and (65) are equilibrium equations, and Eq. (64) permits writing

$$\sum \left(A_i + C_i (x - x_i)^2\right) = a_0 + a_1 x \tag{66}$$

in Eq. (61), where a_0 and a_1 are new constants, so that Eq. (61) becomes

$$y(x) = a_0 + a_1 x + \sum Q_i |x - x_i|^3 \tag{67}$$

In the case where the spline is symmetrical about the origin, as is the case for the AICs $[C_{hs}(k)]$ and $[C_{hDh}(k)]$ which are symmetrical functions of the reduced frequency k, we may use the method of images and Eq. (67) becomes

$$y(x) = a_0 + \sum Q_i \left(|x - x_i|^3 + |x + x_i|^3\right) \tag{68}$$

and only Eq. (63) needs to be satisfied in addition to Eq. (68). The matrix form for Eqs. (63) and (68) in terms of the specified values of $y(x_j)$ is

$$\begin{Bmatrix} 0 \\ y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 & \cdots & 1 \\ 1 & K_{11} & K_{12} & K_{13} & \cdots & K_{1N} \\ 1 & K_{12} & K_{22} & K_{23} & \cdots & K_{2N} \\ 1 & K_{13} & K_{23} & K_{33} & \cdots & K_{3N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \\ 1 & K_{1N} & K_{2N} & K_{3N} & & K_{NN} \end{bmatrix} \begin{Bmatrix} a_0 \\ Q_1 \\ Q_2 \\ Q_3 \\ \vdots \\ Q_N \end{Bmatrix} \tag{69}$$

where $K_{ji} = |x_j - x_i|^3 + |x_j + x_i|^3$ (70)

The left hand side of Eq. (69) may be written

$$\begin{Bmatrix} 0 \\ y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_N \end{Bmatrix} \tag{71}$$

The matrix form for an interpolated value of $y(x_k)$ is

$$y_k = \lfloor 1 \ K_{k1} \ K_{k2} \ \cdots \ K_{kN} \rfloor \begin{Bmatrix} a_0 \\ Q_1 \\ Q_2 \\ \vdots \\ Q_N \end{Bmatrix} \tag{72}$$

where

$$K_{kj} = |x_k - x_j|^3 + |x_k + x_j|^3 \tag{73}$$

Combining Eqs. (69), (71) and (72) leads to the desired interpolation coefficients I_{kj} defined by

$$y_k = \lfloor I_{kj} \rfloor \{y_j\} \tag{74}$$

where $\{y_j\}$ denotes

$$\begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{Bmatrix}$$

and

$$\lfloor I_{kj} \rfloor =$$

$$\lfloor 1 \;\; K_{k1} \;\; K_{k2} \;\; \cdots \;\; K_{kN} \rfloor \begin{bmatrix} 0 & 1 & 1 & 1 & \cdots & 1 \\ 1 & K_{11} & K_{12} & K_{13} & \cdots & K_{1N} \\ 1 & K_{12} & K_{22} & K_{23} & \cdots & K_{2N} \\ 1 & K_{13} & K_{23} & K_{33} & \cdots & K_{3N} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & K_{1N} & K_{2N} & K_{3N} & \cdots & K_{NN} \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \tag{75}$$

Equation (74) provides the linear spline interpolation scheme needed to obtain the AICs in the frequency lining up process. The inverse required in Eq. (75) is ill-conditioned for a large number N of interpolated points, say N=50; however, a large number are not needed in the flutter analysis and N will be limited to $N \leq 10$ in the present applications.*

*A lower limit, say $N \leq 5$ is adequate for most applications unless an extremely wide frequency spectrum must be covered.

SECTION III

AERO-SERVO-ELASTIC STABILITY ANALYSIS PROGRAM: MPASES

Program Description

MPASES is a general purpose digital computer program for the analysis of the closed-loop stability problem. This program, which is a modification of PASES (Ref. 1), formulates the problem by combining a classical method for determining servomechanism system stability with the more realistic British method of flutter analysis in lieu of the American method used in PASES. With the input of an arbitrary number of elastic degrees of freedom, aerodynamic influence coefficients and the control system description, the stability of a missile or aircraft configuration as an aero-servo-elastic system can be investigated. When no aerodynamics are involved, the servo-elastic system stability can be determined.

Dynamic storage allocation utilized throughout MPASES provides the most efficient use of core storage through the variable dimensioning of all arrays. Because the program is based on the theory that permits the inclusion of the aerodynamic forces as spring and damper terms in the equations of motion rather than as complex inertial terms, only real matrices are used in the eigenvalue solution. This makes for additional economy in computer usage.

MPASES uses the method of revising the aerodynamics by lining up the frequencies and performing the spline interpolation of the aerodynamic terms necessary in the iterative eigenvalue solution. This process is repeated until the required number of modes and eigenvectors are obtained for a particular velocity. Moreover, if the analysis Mach number (velocity determining factor) differs from the AIC Mach numbers by more than a specified 'deviation' value (input by the submitter), spline interpolation of the

aerodynamic forces is performed also. In this case, the program interpolates for Mach number <u>before</u> the reduced frequency interpolation for each mode is carried out.

The variables in the stability analysis of an aero-servo-elastic system are servo-gains, velocities (through Mach number and speed of sound input) and altitude (air density input). An option is provided to vary the gain of a single servo component or control surface with the density and Mach number held constant. When this option is executed, the coefficients of the <u>output</u> of the desired servo element or control surface are <u>divided</u> by the gain factor, K. This method facilitates programming, since there may be more than one input.

Since dynamic dimensioning is used, the program limitations are minimal. Stability results can be obtained for up to five (5) altitudes and for each density, as many as ten (10) different velocities can be analyzed. In addition, twenty-five (25) gain variations are permitted. The number of AIC matrices that can be input is limited to ten (10) reduced frequencies for each of five (5) Mach numbers. There are <u>no</u> size restrictions for any of the other parameters existing in the program.

The total number of memory units (words or bytes) required to execute MPASES is completely dependent upon input. The size of the program is reflected in the length of the blank common block found in the 'MAIN' section of the program. The length can readily be altered to accommodate different analyses. Likewise, dimensions of the arrays limiting the program as stipulated above can be changed to the desired size.

Subroutine Description

MAIN	Main program for MPASES. Reads and prints basic input data. Allocates dynamic storage for all arrays. Calling program for subroutines MASS, SERVØ, FØRMAB, AIC, MPSE and MPAS.
AIC	Reads and prints complex AIC matrices. Separates AIC's into real and imaginary parts. Formulates generalized aerodynamic forces and stores on temporary File 1. Calls subroutine GENMAT.
EGNVCT	Finds the eigenvector from a real matrix for which the complex eigenvalue is known. Uses the inverse power method with shifts. The eigenvector obtained is complex with the largest value normalized to unity. A singular matrix returns an eigenvector of zeros.
EIGRF	Computes eigenvalues of a real, triangular matrix. Calls routines from the IMSL Package (see Ref. 7). Called by subroutine SOLV.
FØRMAB	Generates the A and B matrices which constitute the eigenvalue problem. Stores [A] and [B] on temporary File 2.
GENMAT	Formulates the generalized mass and aerodynamic force matrices utilizing rigid body, control surface and vibration modes. Calls subroutine MATMPL. Called by subroutines AIC and MASS.

INTERP Generates the constant portion of the linear spline interpolation coefficients matrix.

Prints message if spline interpolation matrix is singular.

Calls subroutine INVERS.

INVERS Finds the inverse of a real, square matrix.

Called by subroutines INTERP and SØLV.

MASS Reads and prints the weight matrix in pounds. Converts weight to mass units (slugs).

Reads and prints rigid body control surface modes, free-free vibration modes and restrained (rigid body) modes.

Stores modal data on temporary File 4.

Reads and prints damping coefficients and vibration frequencies (Hertz). Converts frequencies to rps units.

Formulates generalized mass matrix.

Generates generalized stiffness and damping matrices if flexible modes are input.

Calls subroutine GENMAT.

MATMPL Multiplies real, two-dimensional matrices.

If requested, transposes post-multiplier matrix to perform as a pre-multiplier.

Called by subroutines INTERP, SERVØ, GENMAT and SØLV.

MPAS Obtains stability results from aero-servo-elastic analysis for altitudes and velocities requested.

Reads Mach numbers at each altitude for which analysis is to be made.

Calculates velocity (from Mach number and speed of sound) and constants for the aerodynamic forces.

Performs Mach number interpolation if necessary.

Reads File 2 - brings [A] and [B] into core.

Executes gain option if requested; stores A and B matrices reflecting gain factor on tempoerary File 3.

MPAS (Cont.) Carries out spline interpolation of the aerodynamic forces for reduced frequency for each mode.

Generates new A and B matrices with changed aerodynamics for the particular mode.

Calls subroutine SØLV for eigenvalue solution of each mode.

Prints stability results.

Prints eigenvectors, if requested, for the analysis.

Reads File 4.

Computes and prints structural deflections at system mass points for each mode.

Calls subroutines INTERP, SPLINE and SØLV.

MPSE Obtains stability results from servo-elastic analysis for number of modes requested.

Reads File 2 - places [A] and [B] into core.

Executes gain option if requested.

Calls subroutine SØLV for eigenvalue solution of required modes.

Prints stability results.

Prints eigenvectors for each mode.

Note that the eigenvectors are <u>always</u> computed and printed for a servo-elastic analysis.

Reads File 4.

Computes and prints structural deflections at system mass points for each mode.

Calls subroutine SØLV.

RESULT Forms the array where stability results for each mode are stored.

Calls subroutine EGNVCT to compute the eigenvectors (only upon request for aero-servo-elastic analysis, always for servo-elastic analysis).

Called by subroutine SØLV.

SERVØ Reads coefficients from servo differential equations.

Prints control system description.

Computes input to rate gyros and accelerometers from differentiation or interpolation matrices.

Determines maximum order of servo elements.

Prints pertinent information concerning all servo elements.

SERVØ (Cont.)	Generates [X2], [X1], [X0], [Y1], [Y0], [Z0].
	Calls subroutine MATMPL.
SØLV	Prints the A and B matrices.
	Generates the dynamic matrix (eigenvalue problem).
	Calls subroutine EIGRF for eigenvalue solution.
	"Lines-up the frequencies" in aero-servo-elastic analysis.
	Determines real and oscillatory roots for each mode.
	Estimates new reduced frequencies.
	Calls subroutine RESULT to compute stability results and eigenvector for each mode.
	Calls subroutines MATMPL, INVERS, EIGRF (from IMSL Package), and RESULT.
	Called by subroutines MPAS and MPSE.
SPLINE	Generates the linear spline interpolation matrix.
	Determines the Lagrangian coefficients.
	Used in the frequency lining-up process.
	Called by subroutine MPAS.

Programming Symbols - A partial list of FØRTRAN symbols used in MPASES is presented. Additional symbols are defined in the Input Instructions.

1. Integer Variables

ID	Density index
IE	Mode counter
IG	Gain index
IK	Reduced frequency index
IM	Mach number index
IMØDE	Actual number of modes calculated for stability analysis

IOB	First real root (eigenvalue with positive real part and zero imaginary part)
IOE	Last real root
ISING	Singular matrix control. If ISING = 1, matrix is non-singular; return from subroutine INVERS.
J1	BLANK COMMON length required to call subroutine MASS
J2	BLANK COMMON length required to call subroutine SERVØ
J3	BLANK COMMON length required to call subroutine FØRMAB
J4	BLANK COMMON length required to call subroutine MPSE or subroutine AIC
J5	BLANK COMMON length required to call subroutine MPAS
JTØT	Largest values of J1, J2, J3 and J4 for servo-elastic analysis; or largest value of J1, J2, J3, J4 and J5 for aero-servo-elastic analysis. JTØT is the minimum BLANK COMMON length required to execute MPASES.
KØP	Column assignment in A and/or B matrices for zero and first order servos.
KØ1	Column assignment in A and/or B matrices for second order servo-velocity.
KØ2	Column assignment in A and/or B matrices for second order servo-displacement.
KXM	Control for Mach number interpolation.
K1	Order of servo element for which the gain varies.
K2	Column assignment in the X, Y and/or Z arrays for servo whose gain varies.
L	Counter for coefficients from servo differential equations including inputs to control surfaces, outputs from servos, and inputs to servos.
MDF	Number of mass points; used as a variable dimension involving flexible modes. (MDF = 1 when NFM = 0)
MDR	Number of mass points; used as a variable dimension involving rigid body modes. (MDR = 1 when NRM = 0)
MFM	Number of flexible modes; used as a variable dimension. (MFM = 1 when NFM = 0)

MRM	Number of rigid body modes; used as a variable dimension. (MRM = 1 when NRM = 0)
M1	Number of flexible modes <u>or</u> rigid body modes, whichever is larger.
M2	M1 <u>or</u> the number of control surfaces, whichever is larger.
M3	Number of reduced frequencies <u>or</u> Mach numbers, whichever is larger. (AIC input)
M31	M3 + 1.
NFR	Sum of the number of flexible modes and rigid body modes.
NIO	Counter for real roots.
NK1	NK + 1
NMØDE	Sum of the number of flexible modes, rigid body modes and control surfaces.
NM1	NM + 1
NPART	Number of partitions in AIC matrix. (NPART = NA if NA > 1)
NPR	Counter for number of actual modes in the servo-elastic analysis.
NRO	Control for determining the number of real roots.
NSERV	Total number of coefficients from servo differential equations including inputs to control surfaces, outputs from servos, and inputs to servos; used as a variable dimension.
NT	Sum of the number of flexible modes, rigid body modes, control surfaces and <u>all</u> servo elements.
NTC	Sum of the number of flexible modes, rigid body modes, control surfaces and second order servo elements.
NTT	Order of the eigenvalue problem, 2*NTC+N0+N1.
NVEC	Control for calculation and printing of eigenvectors.
N0	Number of zero order servos
N1	Number of first order servos
N2	Number of second order servos

2. <u>Real Variables</u>

ANØRM	Test for normalizing structural deflections.
CØNAI	Constant for generalized aerodynamic forces-imaginary part, $-\frac{1}{2}\rho VS$.
CØNAR	Constant for generalized aerodynamic forces-real part, $-\frac{1}{2}\rho V^2 S/\bar{c}$.
CØNI	Constant for imaginary part of AIC matrix, $k(\bar{c}s/S)$.
CØNR	Constant for real part of AIC matrix, $2k^2(\bar{c}s/S)$.
DET	Value of determinant returned from subroutine INVERS if matrix is non-singular.
EK	Reduced frequency, $b_r\omega/V$, used in the iterative procedure; determined by the estimated frequency, ω.
EW	Estimated frequency for successive iterations of the eigenvalue solution; determined by the 'lining-up the frequencies' process.
VEL	Velocity for the stability analysis, ft/sec.
VELK	Velocity for the stability analysis, knots.
XNØRM	Normalizing factor for structural deflections.

3. <u>Integer Arrays</u> (Variable dimensions indicated in parentheses)

IANA(NTT)	Used in subroutine SOLV as an argument when subroutine RESULT is called. This vector is eventually used in EGNVCT as L1 described below.
INDEX(NTT,3)	Used in subroutine INVERS for working storage.
KØ(NSE,2)	Stores information concerning the <u>maximum</u> order of all servo elements and their column assignments in the X, Y and Z arrays.
L1(NTT)	Used in subroutine EGNVCT to restore order of the elements in the eigenvector.
L2(NTT)	Used in subroutine EGNVCT as working storage.
NL(NSERV,2)	Stores information concerning the number of the servo element from which there is input and output.
NØR (NSE,NSERV)	Stores information concerning the order of the servo element coefficients.

4. <u>Real Arrays</u> (Variable dimensions indicated in parentheses)

A(NTT,NTT)	Matrix in the eigenvalue problem formulation.
AFI (NFR,NMØDE,NK)	Generalized aerodynamics used for reduced frequency spline interpolation - imaginary part.
AFR (NFR,NMØDE,NK)	Generalized aerodynamics used for reduced frequency spline interpolation - real part.
AMI (NFR,NMØDE,NM)	Generalized aerodynamics used for Mach number spline interpolation - imaginary part.
AMR (NFR,NMØDE,NM)	Generalized aerodynamics used for Mach number spline interpolation - real part.
B(NTT,NTT)	Matrix in the eigenvalue problem formulation.
BINV(NTT,1)	Column of constants used in subroutine INVERS.
BC(NTT,NTT)	$\gamma_o[A] + [B]$, where γ_o is the eigenvalue shift.
C(NTT,NTT)	Dynamic matrix used for the eigenvalue solution.
CF(MFM)	Structural damping matrix in its equivalent viscous form (diagonal)
CHDH(NDF,NDF)	$k(\bar{c}s/S)[C_{hI}]$, where $[C_{hI}]$ is the imaginary part of the AIC matrix $[C_h]$.
CHS(NDF,NDF)	$2K^2(\bar{c}s/S)[C_{hR}]$, where $[C_{hR}]$ is the real part of the AIC matrix $[C_h]$.
DFM(1,MFM)	$-K[D][h_F]$, where K is the rate gyro or accelerometer gain, D is the differention or interpolation vector, and h_F are the flexible modes.
DRM(1,MRM)	$-K[D][h_R]$, where K is the rate gyro or accelerometer gain, D is the differentiation or interpolation vector, and h_R are the rigid body modes.
FRC(M2,M2)	Partition of generalized mass or aerodynamic matrix used in subroutine GENMAT.
GAI(NFR,NMØDE)	Matrix of generalized aerodynamics as output from subroutine GENMAT - imaginary part.
GAR(NFR,NMØDE)	Matrix of generalized aerodynamics as output from subroutine GENMAT - real part.

GENM (NFR,NMØDE)	Generalized mass matrix
P(M31)	Row vector for spline interpolation used in subroutine SPLINE.
PSK(NK)	Vector of LaGrangian coefficients from reduced frequency spline interpolation
PSM(NM)	Vector of LaGrangian coefficients from Mach number spline interpolation
RI(NTT)	Imaginary part of eigenvalue
RR(NTT)	Real part of eigenvalue
SAFI (NFR,NMØDE)	Matrix of generalized aerodynamic forces for estimated reduced frequency obtained from spline interpolation - imaginary part.
SAFR (NFR,NMØDE)	Matrix of generalized aerodynamic forces for estimated reduced frequency obtained from spline interpolation - real part.
SI(M31,M3)	Matrix of 'ones' for spline interpolation - used in subroutine INTERP.
SM(NM1,NM)	Constant spline interpolation matrix for Mach numbers.
SMK(M31,M31)	Matrix of Mach numbers or reduced frequencies in the constant portion of the spline interpolation matrix - used in subroutine INTERP.
SK(MFM)	Generalized stiffness matrix (diagonal).
SPK(NK1,NK)	Constant spline interpolation matrix for reduced frequencies.
STAB(MØDE,6)	Stores stability results for each mode.
TM(M1,NDF)	Intermediate array used in subroutine GENMAT.
XMØDE (NDF,NMØDE)	Matrix of flexible, rigid body and control surface modes.
X0(NT,NT)	Coefficient matrix of second order variables, displacement.
X1(NT,NT)	Coefficient matrix of second order variables, velocity.
X2(NT,NT)	Coefficient matrix of second order variables, acceleration.

Y0(NT,NSE)	Coefficient matrix of first order variables, displacement.
Y1(NT,NSE)	Coefficient matrix of first order variables, velocity.
Z0(NT,NSE)	Coefficient matrix of zero order variables, displacement.
Z	Dynamic dimensioning array in BLANK COMMON.

5. Complex Arrays (Variable dimensions indicated in parentheses)

C2(NTT)	Complex working storage for subroutine EGNVCT.
C3(NTT)	Complex eigenvector for a particular mode, computed by subroutine EGNVCT.
DEFL (NDF)	Structural deflections for the system mass points (determined by input modal data and eigenvectors).
E(NTT)	Complex eigenvalues resulting from the eigenvalue solution in subroutine EIGRF.
U(NTT,NTT)	$\gamma[A] + [B]$, where γ is the complex eigenvalue for a particular mode; used in subroutine EGNVCT to determine eigenvector.
V(1,1)	Psuedo complex eigenvector storage in subroutine EIGRF (not used).
VEC(MØDE,NTT)	Stores eigenvectors for each mode (complex).
WK(NTT)	Work area in subroutine EIGRF.

Processing and Programming Considerations

1. Operation

Standard FØRTRAN IV processor system. Operable on the CDC computer; model 6600, Cyber 175-Scope 3.4.3 system.

Note: MPASES can be made operable on all computer systems with minor modifications. Probable necessary changes are listed below:

Deletion of PRØGRAM statement at beginning of 'MAIN' section of program.
END-ØF-FILE (EØF) statement alterations.
Alphanumeric modifications dependent upon the number of characters per word in the operating system used.
Single precision to double precision accuracy. (MPASES, as presented in this report, has single precision accuracy.)

2. Core Storage

Number of memory units (words, bytes, etc.) required to execute is dependent upon input data reflected in the length of BLANK

COMMON. Estimate of BLANK COMMON length can be accomplished as follows:

Servo-Elastic Analysis:

$$LENGTH = 1 + NTT(4NTT + 2MØDE + 9) + NDF(NMØDE + 2) + 6MØDE$$

OR

$$LENGTH = 1 + NDF(NDF+NMØDE + M1) + NFR(NMØDE) + 4NFM + (M2)^2$$

Use the larger of the above two estimates.

Aero-Servo-Elastic Analysis:

$$\begin{aligned} LENGTH = 11 &+ NTT(4NTT + 2MØDE + 9) \\ &+ 2(NFR)(NMØDE)(NK + NM + 1) + 2(M3 + 1)^2 \\ &+ NM(NM + 2) + NK(NK + 2) + NDF(NMØDE + 2) \\ &+ 6MØDE \end{aligned}$$

OR

$$LENGTH = 1 + NDF(4NDF + NMØDE + M1) + 2(NFR)(NMØDE) + (M2)^2$$

Use the larger of the two. If a number greater than NTT was assigned to the parameter 'MODE,' use MODE=NTT in the above calculations. Note that the first estimate for each analysis is eigenvalue problem size oriented, while the second is dependent upon the number of system mass points.

Refer to this section under Programming Symbols for definitions.

Actual BLANK COMMON length required for the analysis follows the stability results in the program's printed output. This information will enable the user to make more efficient use of core storage in subsequent analyses by recompiling the 'MAIN' section with a more realistic BLANK COMMON length.

Note. MPASES, as presented in this report, contains a BLANK COMMON length of 10,000. This length was more than adequate to perform the analyses in the sample problems used as examples. (see Section 4.0.)

3. Auxiliary Files

Standard input read file (5).

Standard output print file (6).

Four temporary utility files (1, 2, 3, and 4).

Note: To avoid the cumbersome handling of data cards, it is suggested that data sets be generated containing the AIC's, modal data, etc. Consequently, MPASES can be modified to accept this input from auxiliary files in lieu of cards.

Input Instructions

Units - All units are taken in the pound-feet-second system with the exception of density, which must be in slugs/cubic feet. The weight, which is input in pounds, is converted to slugs internally in the program. Likewise, MPASES converts the frequency from Hertz to rps units within the program. The AIC's must be non-dimensional when input with no altitude consideration (i.e., $\rho=1$).

Data Deck Setup

1. Title cards (2).
2. Control card describing data input.
3. Eigenvalue shift, γ_0.
4. Geometric properties of air vehicle (s, $\bar{c}$, S) and Mach number deviation (DM).
5. Mach numbers and reduced frequencies, i.e., for AIC input.
6. Density and speed of sound for each altitude.
7. Number of velocities to be analyzed for each altitude.
8. Gain factors, if gain option is to be exercised.
9. Weight matrix.
10. Rigid body control surface modes.
11. Structural damping coefficients if NFM > 0.
12. Frequencies of vibration (flexible) modes if NFM > 0.
13. Flexible modes, if any.

14. Restrained (rigid body) modes, if any.

15. Coefficients of differential equations describing the servo elements in the control system.

16. AIC matrices.

17. Mach numbers to determine velocities to be analyzed at each altitude.

Items 4, 5, 6, 7, 16 and 17 are included for aero-servo-elastic analysis only.

Detailed description of input data follows.

Input Data Description

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION
1.	TITLE	18A4	1-72		Any alphanumeric statement.
2.	TITLE	18A4	1-72		Any alphanumeric statement.
			NOTE:	Two cards must be input. May be blank.	
3.	CONTROL	14I5	1-5	MCØDE	= 1, diagonal weight matrix input. = 2, coupled weight matrix input.
			6-10	NDF	Number of mass points in the complete system - degrees of freedom (NDF > 0).
			11-15	NFM	Number of flexible modes (NFM $\geq$ 0). If NRM = 0, NFM > 0).
			16-20	NRM	Number of rigid body modes (NRM $\geq$ 0). If NFM = 0, NRM > 0).
			21-25	NC	Number of rigid body control surface modes; i.e., number of control surfaces (NC > 0).
			26-30	NSE	Total number of servo elements in the control system, including rate gyros and accelerometers (NSE > 0).

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION
			31-35	NA	= 0, no aerodynamics, i.e., servo-elastic analysis only. > 0, AIC input, i.e., aero-servo-elastic analysis. Input as follows: NA = number of partitions contained in each AIC matrix.
			NOTE:		NA = 1, AICs derived from Doublet-Lattice, Mach Box theories, etc. NA > 1, AICs derived from Strip, Piston Theories, etc. (NA = number of strips). It is assumed that <u>all</u> AICs are obtained from the <u>same</u> method for any <u>one</u> aero-servo-elastic analysis.
			36-40	NM	Number of Mach numbers for which there is AIC input ($NM \leq 5$). If NA = 0, NM = 0.
			41-45	NK	Number of reduced frequencies (k) for each Mach number ($NK \leq 10$). If NA = 0, NK = 0.
			NOTE:		The reduced frequencies <u>must</u> be the same for each Mach number.
			46-40	ND	Number of altitudes ($ND \leq 5$). If NA = 0, ND = 0.
			51-55	MØDE	Number of modes requested for the stability analysis. Use a <u>large</u> number if <u>all</u> modes inherent to the analysis are to be considered.
			NOTE:		Due to the nature of the 'lining up the frequencies' method of analysis, the eigen-value solution <u>may</u> result in <u>less</u> modes than requested.
			56-60	NG	= 1, <u>no</u> gain variation. > 1, gain option executed. Input as follows: NG = number of gain variations (K). ($1 \leq NG \leq 25$)
			61-65	NGS	= 0, <u>no</u> gain variation. > 0, NGS = number of the particular <u>servo element</u> for which the gain varies. < 0, NGS = number of the <u>control surface</u> for which the gain varies.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION
			NOTE:	The gain factor (K) can vary for only one servo element or control surface in any one stability analysis.	
			66-70	NAB	= 0, A and B matrices not printed. = 1, A and B matrices printed by rows.
			NOTE:	If NA = 0, [A] and [B] as generated for the analysis are printed. If NA > 0, [A] and [B] as generated for k=0 only are printed.	
4.	SHIFT	E12.0	1-12	GAMMA	Shift eigenvalue (γ_o). Choice of γ_o is left to the user.
			NOTE:	See Section 2 (Theoretical Development) and Section 4 (Sample Problems) for details.	

If NA = 0, OMIT the following cards: 5, 6, 7, 8 and 9.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION
5.	AERØ CØNSTANTS	4E12.0	1-12	SS	Semi-span (s), feet.
			13-24	CBAR	Mean aerodynamic chord ($\bar{c}$), feet.
			25-36	S	Surface area (S), sq.ft.
			37-48	DM	Mach number deviation. Choice of DM is left to the user.
			NOTE:	If the difference between the Mach number for which the analysis is to be performed and the Mach number for which there is AIC input is greater than DM, Mach number interpolation of the AICs will be made. If NM = 1, no interpolation.	
					If NA = 0, OMIT Card 5.
6.	AIC DATA	5E12.0		XM(I)	Mach number; I = 1, NM. If NA = 0, OMIT CArd 6.
7.	AIC DATA	6E12.0		XK(I)	Reduced frequency, k; I=1, NK. Continue on next card if necessary. If NA = 0, OMIT Card 7.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION
8.	AERØ DATA	6E12.0	1-12	DENS(1)	Air density, ρ , associated with the first altitude at which the analysis is to be made; slugs/cu.ft.
			13-24	SØS(1)	Speed of sound associated with the first altitude; ft/sec.
			25-36	DENS(2)	Density for second altitude.
			37-48	SØS(2)	Speed of sound for second altitude.
			. . .	. . .	. . .
					DENS(I), SØS(I); I=1, ND. Continue on next card if necessary.
					If NA = 0, OMIT Card 8.
9.	AERØ DATA	5I5		MD(I)	Number of velocities (determined by Mach number input - see Card 26) for each altitude; I = 1, ND. (MD ≤ 10)
					If NA = 0, OMIT Card 9.
10.	GAIN	6E12.0		GAIN(I)	Gain factor, K. I = 1, NG. Continue on next card if necessary. (NG ≤ 25)
					If NGS = 0, OMIT Card 10.
11.	WEIGHT	6E12.0		WT(I,I)	If MCØDE = 1, vector of diagonal elements of weight matrix, pounds. I = 1, NDF. Continue on next card if necessary.
				WT(I,J)	If MCØDE = 2, elements of upper triangle of coupled weight matrix, lbs. Input by rows; each row starts on a new card. I = 1, NDF, J = I, NDF. Continue on successive cards to complete each row.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION
12.	MØDE	6E12.0		CMØDE(J,I)	Rigid body control surface modes. Each mode starts on a new card. I = 1, NC, J = 1, NDF. Continue on successive cards to complete each mode.
13.	DAMPING	6E12.0		GF(I)	Structural damping coefficient corresponding to vibration mode (I). I = 1, NFM. Continue on next card if necessary. If NFM = 0, OMIT Card 13.
14.	FREQUENCY	6E12.0		FREQ(I)	Frequency of vibration mode (I), Hz. I = 1, NFM. Continue on next card if necessary. If NFM = 0, OMIT Card 14.
15.	MØDE	6E12.0		FMØDE(J,I)	Free-free vibration (flexible) modes. Each mode starts on a new card. I = 1, NFM, J = 1, NDF. Continue on successive cards to complete each mode. If NFM = 0, OMIT Card 15.
16.	MØDE	6E12.0		RMØDE(J,I)	Restrained (rigid body) modes. Each mode starts on a new card. I = 1, NRM, J = 1, NDF. Continue on successive cards to complete each mode. If NRM = 0, OMIT Card 16.

Input description of the servo differential equation coefficients follows:

IMPORTANT: All servo differential equations must be equated to zero, i.e., all terms should be on one side of the equation, before attempting to input the coefficients.

NOTE: Reference to servo elements and control surfaces is by number; therefore, numbers should be assigned to each servo element (1 to NSE) and control surface (1 to NC). Servo elements also include the rate gyros and accelerometers in the control system.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION

REPEAT the following cards 17, 18 and 19 for each control surface, i.e., NC times.

17. Output from Control Surface. FORMAT (3E12.0)

Columns	1-12	X0(I)	Zero order coefficient for Control Surface (I).
	13-24	X1(I)	First order coefficient for Control Surface (I).
	25-36	X2(I)	Second order coefficient for Control Surface (I). Note: I=1, NC.

18. Input Control Card. FORMAT (I5)

Columns	1-5	INC	Number of inputs from servo elements to control surface (I).

REPEAT card 19 for input from each servo element to Control Surface (I), i.e., INC times.

19. Input from Servo Element. FORMAT (2I5,2X,3E12.0)

Columns	1-5	K	Servo element from which there is input.
	6-10	NØ	Order of servo element coefficients.
	13-24	C0	Zero order coefficient.
	25-36	C1	First order coefficient.
	37-48	C2	Second order coefficient.

If NØ = 0, zero order; only C0 input.
= 1, first order; C1 must be input (C0 may be zero.
= 2, second order; C2 must be input (C0 and C1 may be zero).

REPEAT the above card 19 INC times.

REPEAT the above cards 17, 18 and 19 NC times.

REPEAT the following cards 20, 21 and 22 for each servo element, i.e., NSE times (not necessarily in sequential order).

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION

20. Output from Servo Element. FORMAT (2I5,2X,3E12.0)

Columns	1-5	I	Servo element number from which there is output.
	6-10	NØ	Order of servo element coefficients.
	13-24	C0	Zero order coefficient
	25-36	C1	First order coefficient.
	37-48	C2	Second order coefficient.

If NØ = 0, only C0 is input.
= 1, C1 must be input (C0 may be zero).
= 2, C2 must be input (C0 and C1 may be zero).

21. Input Control Card. FORMAT (I5)

Columns	1-5	INS	Number of inputs from servo elements (or control surfaces) to servo element (I).

If there are no inputs to Servo Element (I), INS = 0.

REPEAT card 22 for input from each servo element or control surface to Servo Element (I), i.e., INS times.

If INS = 0, OMIT Card 22.

22. Input from Servo Element or Control Surface. FORMAT (2I5,2X,3E12.0)

Columns	1-5	K	Servo element number from which there is input.
		-K	Control surface number from which there is input.
	6-10	NØ	If K > 0, NØ = order of servo element coefficients. If K < 0, NØ = order of control surface coefficients.

NØ = -1, input from body angular rate to rate gyro.
= -2, input from body acceleration to accelerometer.

NOTE: If NØ < 0, K = 0.

13-24	C0	Zero order coefficient.
25-36	C1	First order coefficient.
37-48	C2	Second order coefficient.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION

If NØ = 0, only C0 input.
= 1, C1 must be input (C0 may be zero)
= 2, C2 must be input (C0 and C1 may be zero).
= -1, only C0 input, where
C0 = $-K_g$ (rate gyro gain)
= -2, only C0 input, where
C0 = $-K_a$ (accelerometer gain)

NOTE: When NØ < 0, Card 22 must be followed by Card 23.

23. If NØ = -1,
Differentiation Row Vector used to determine angular velocity for the rate gyro.

If NØ = -2,
Interpolation Row Vector used to describe body acceleration for the accelerometer.

FORMAT 6E12.0 D(I) Element of differentiation or interpolation row matrix. I = 1, NDF.
Continue on next card if necessary.

NOTE: See Ref. 1 for description.

REPEAT the above Card 22 (and Card 23, if applicable) INS times.

REPEAT the above Cards 20, 21 and 22 NSE times.

Input instructions for the AIC matrices $[C_h]$ follow:

NOTE: It is assumed that the AIC's are dimensionless and do not reflect density (i.e., $\rho = 1$).

If NA = 0, OMIT Cards 24 and 25.

If NA = 1, input only Card 25.

If NA > 1, input Cards 24 and 25.

The AICs are input for all the reduced frequencies, i.e., for XK(I), where I = 1, NK for each Mach number XM(J), where J = 1, NM.

NOTE: The reduced frequencies, k, must be the same for each Mach number.

If NA = 1, REPEAT Card 25 for each AIC matrix, i.e., (NK*NM) times.

If NA > 1, REPEAT Cards 24 and 25 NA times for each AIC matrix. See Card 3. NA = Number of partitions (NPART) in the AIC matrix. All the input AIC's must have the same number of partitions.

REPEAT until all AIC's are input, i.e., (NK*NM) times.

24. Partition Control Card. FORMAT (2I5)

Columns 1-5 MS Size of partition.

NOTE: The sum of the sizes (MS) of all the partitions in each AIC matrix must equal NDF.

6-10 NZERØ = 1, all elements of partitions are equal to zero. Do not input. OMIT Card 25.

= 0, non-zero partition. Input Card 25.

25. AIC Partition or Matrix. FORMAT (6E12.0)

CH(I,J) If NA = 1, complete AIC matrix (complex).
I = 1, NDF, J = 1, NDF.

If NA > 1, AIC partition (complex).
I = 1, MS, J = 1, MS.

Input by rows. Each row starts on a new card. Continue on successive cards to complete each row.
The imaginary part of each element follows the real part, e.g.:

Columns	1-12	CH(1,1)	Real
	13-24	CH(1,1)	Imaginary
	25-36	CH(1,2)	Real
	37-48	CH(1,2)	Imaginary
	.	.	
	.	.	
	.	.	

OMIT the following Card 26, if NA = 0.

REPEAT Card 26 for each altitude, i.e., ND times.

NO.	CARD	FORMAT	COLUMNS	FORTRAN NAME	DESCRIPTION

26. Mach Numbers for Analysis. FORMAT (6E12.0)

XMACH(I) Mach numbers selected for stability analysis at a particular altitude. I = 1, NMACH, where NMACH is the number of velocities (determined by the Mach number, XMACH(I) and the speed of sound, SØS(J) for altitude J), i.e., NMACH=MD(J). Refer to Card 9. (NMACH $\leq$ 10). Continue on next card if necessary. No eigenvectors are obtained or printed with the stability analysis when XMACH(I) > 0.

-XMACH(I) Same as above.
Complex eigenvectors are calculated and printed for each mode when XMACH(I) < 0.

NOTES: XMACH(I) may be the same or different for each altitude. Do not confuse XMACH with XM which are the Mach numbers for the input AIC's.

REPEAT above Card 26 ND times.

Program Output Description

Input Data

1. Upper triangle of weight matrix (lbs.).

2. Damping coefficients and frequencies (if free-free vibration modes are present).

3. Mode shapes - flexible, rigid body and control surface.

4. Control system description - coefficients from servo differential equations, rate gyro gains and/or accelerometer gains.

5. Maximum order of each servo element (determined internally in the program).

6. Rows and columns (assigned by the program) in the A and/or B matrices for the coefficients of each servo element.

7. Eigenvector element associated with each servo element (velocity and/or displacement).

8. AIC matrix (by rows) for each reduced frequency and Mach number, if aero-servo-elastic analysis.

A and B Matrices (if requested)

1. For an aero-servo-elastic analysis, only [A] and [B] for k=0 are printed.

2. For a servo-elastic analysis, [A] and [B] as generated for the analysis are printed.

Stability Analysis Results

The results are identified and tabulated for each gain factor variation (if any). In addition, when aerodynamics are included, the results are printed for the velocities analyzed at each altitude.

1. For each mode calculated, the following is printed in tabulated form:

Complex eigenvalue, γ - real part, μ, 1/sec.

Complex eigenvalue, γ - imaginary part, ω, rad/sec.

Damped frequency, Hz.

Undamped frequency, Hz.

Reduced frequency, k, used to determine eigenvalue problem for particular mode.

Fraction of critical damping, ζ.

Time to half amplitude, sec.

2. Complex eigenvectors corresponding to each eigenvalue, if requested for the aero-servo-elastic analysis of a particular velocity. The eigenvectors are always printed with the stability results of a servo-elastic analysis.

3. Structural deflections (complex) representing the system mass points for each mode are printed whenever eigenvectors are calculated.

BLANK CØMMØN Length

The minimum BLANK CØMMØN length required to execute the program. This length is dependent upon the input data.

SECTION IV

SAMPLE PROBLEMS

As examples of the program capabilities we consider a simplified missile and servo configuration first, Case 1, as a rigid body in vacuo, second, Case 2, as a flexible body in vacuo, and third, Case 3, as a flexible body in an incompressible airstream at sea level. The missile is idealized as a uniform beam and control surface as shown in Fig. 1.

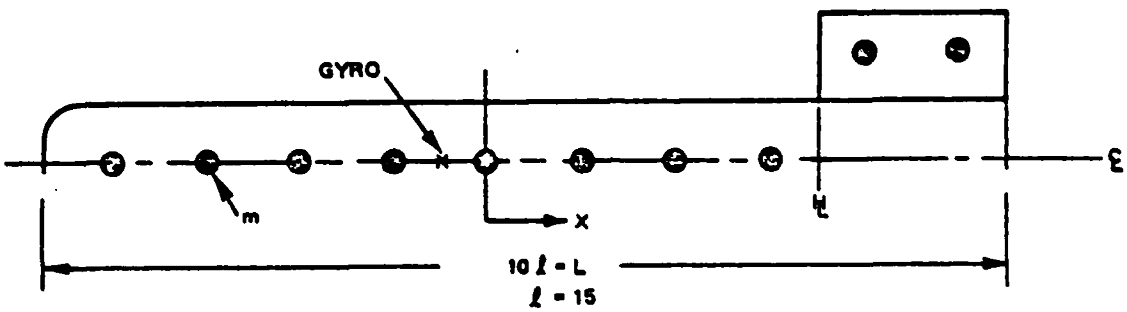

Figure 1. Uniform free-free missile.

The modal characteristics and differentiation matrix for this configuration are given in Ref. 11. For the numerical work we assume the missile to weigh 10m = 1000 lbs., to be 150 inches long, and to have a fundamental frequency of 45 Hz. From the data in Ref. 11, this results in higher frequencies of f_2 = 125.4 Hz and f_3 = 248.2 Hz; only three modes will be considered. The structural dampings in the three modes are assumed to be g_1 = 0.03, g_2 = 0.05, and g_3 = 0.08.

A simple servo system is considered whose block diagram is shown in Fig. 2.

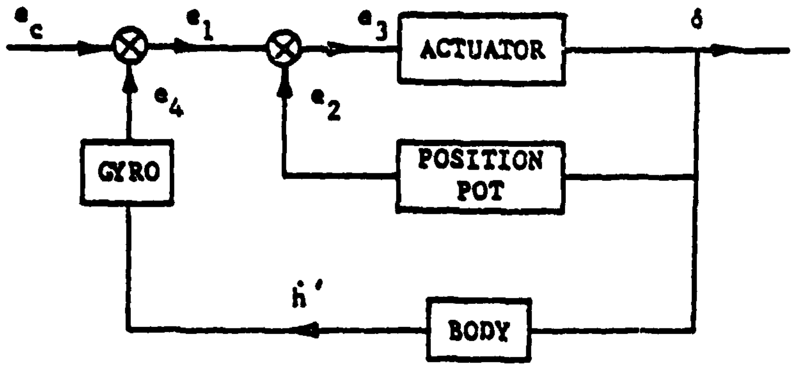

Figure 2. Servo block diagram.

We assume the transfer functions appear as follows.

For the command error,

$$e_1 + e_4 = e_c \tag{76}$$

for the servoposition error,

$$e_1 - e_2 - e_3 = 0 \tag{77}$$

for the actuator,

$$e_3/\delta = K_a s/(T_a s + 1) \tag{78}$$

for the position potentiometer,

$$e_2/\delta = K_p \tag{79}$$

and for the rate gyro,

$$e_4/\dot{h}' = K_g/(s^2/\omega_g^{\,2} + 2\zeta_g s/\omega_g + 1) \tag{80}$$

The differential equations corresponding to Eqs. (78) and (80) are

$$T_a \dot{e}_3 + e_3 - K_a \dot{\delta} = 0 \tag{81}$$

and

$$(1/\omega_g^{\,2})\, \ddot{e}_4 + (2\zeta_g/\omega_g)\, \dot{e}_4 + e_4 - K_g \dot{h}' = 0 \tag{82}$$

The actuator is seen to be a first order servo element and the rate gyro is a second order element. Therefore the second order variables are

$$\{x\} = \begin{Bmatrix} a_F \\ a_R \\ \delta \\ e_4 \end{Bmatrix} \tag{83}$$

the first order variable is

$$\{y\} = \{e_3\} \tag{84}$$

and the zero order variables are

$$\{z\} = \begin{Bmatrix} e_1 \\ e_2 \end{Bmatrix} \tag{85}$$

For the numerical work we assume K_a = 1/6 deg. per deg./sec., T_a = 0.01 sec., K_p = 1 deg./deg., ω_g = 376.991 rad/sec. (60 Hz), ζ_g = 0.70, and K_g = 0.3 deg. per deg./sec. The differentiation matrix for the rate gyro from Ref. 11 is

$$[D] = (1/24\ell)\ [0\ 0\ +1\ -27\ +27\ -1\ 0\ 0\ 0\ 0\ ] \tag{86}$$

where ℓ = 15 inches.

The case descriptions below present the variables in the eigenvectors and discuss the solutions. The data input code sheets and the program printed output for the three sample problems appear following the case descriptions.

Case 1 Description

The first case is the rigid body <u>in</u> <u>vacuo</u>. There are two rigid body modes of plunging and pitching. The eigenvalue calculation is made choosing

$\gamma_o = -1.0$. The $11^{\underline{th}}$ order eigenvector that appears is in the following order:

$$\{V\}^T = \lfloor \dot{a}_{R1}\ \dot{a}_{R2}\ \dot{\delta}\ \dot{e}_4\ a_{R1}\ a_{R2}\ \delta\ e_4\ e_3\ e_1\ e_2 \rfloor \tag{87}$$

There are four zero eigenvalues from the two rigid body modes and four infinite eigenvalues from the command error, the servoposition error, the position potentiometer, and the actuator. The three non-trivial eigenvalues consist of the real actuator damping and the complex conjugate rate gyro frequency and damping. However, since only the eigenvalues with positive frequencies in the conjugate pairs are printed in the stability results, only nine solutions are presented in the printed output. The structural deflections at the ten system mass points for each mode are also printed.

Case 2 Description

The second case is the flexible body *in vacuo*. The addition of three flexible modes to Case 1 results in a $17^{\underline{th}}$ order eigenvector which has the variables printed out in the following order:

$$\{V\}^T = \lfloor \dot{a}_{F1}\ \dot{a}_{F2}\ \dot{a}_{F3}\ \dot{a}_{R1}\ \dot{a}_{R2}\ \dot{\delta}\ \dot{e}_4\ a_{F1}\ a_{F2}\ a_{F3}\ a_{R1}\ a_{R2}\ \delta\ e_4\ e_3\ e_1\ e_2 \rfloor \tag{88}$$

The eigenvalues are obtained by again choosing $\gamma_o = -1.0$. Nine non-trivial eigenvalues are obtained in this case. Three are those obtained in Case 1 and the additional three complex conjugate pairs correspond to the three flexible modes. The stability results present the modes with the frequencies in ascending order, the negative values being omitted. Therefore, only thirteen solutions are printed; the structural deflections for the thirteen modes are also printed.

An unsatisfactory design is seen to exist from the negative damping in the 133.0 Hz mode; the missile, will "buzz" in this mode.

Case 3 Description

The third case is the flexible body in an incompressible flow at sea level with a density $\rho = 0.00237692$ slugs/cu. ft. and velocity $V = 500$ fps. It is the same as Case 2 with the addition of aerodynamic loads. The aerodynamic loads are assumed to act only on the control surface and are derived from the incompressible strip theory presented in Ref. 15. The control surface has an exposed span of 20 inches and a semichord of 15 inches. Five reduced frequencies are chosen for the aerodynamic interpolation, $k = 0.05$, 0.10, 0.20, 0.50, and 1.00; the minimum value of $k = 0.05$ is chosen because of the singularity in the aerodynamic damping at zero frequency when it is computed from Theodorsen's function.

Although there are 17 degrees of freedom in the eigenvalue problems solved, only nine modes can be obtained. This results from the frequency "lining-up" process. The reduced frequencies determined by the estimated frequency in each eigenvalue solution are shown along with the stability results for each mode. In addition, the time to half the amplitude as well as the damping ratio is indicated to facilitate stability evaluation.

The solutions for Case 3 are seen not to be significantly different from those of Case 2. However, we note that the short period mode shows up in place of the rigid body pitching mode. The reduced frequency of the short period mode is $k = 0.03346$ and its damping ratio is $\zeta = 0.1401$. The aerodynamic loads do not cause a large disturbance for the (impractical)

configuration chosen as the example. The servoelastic "buzz" observed in Case 2 is still unstable here. More practical configurations should be studied that have more critical hinge line locations and mass balancing on the control surface.

Keypunch Forms for Cases 1, 2 and 3

IBM FORTRAN Coding Form

```
SAMPLE PROBLEM - CASE 1, 2, OR 3                          TITLE CARD 1        CARD 1
          MISSILE AND SERVO SYSTEM                        TITLE CARD 2        CARD 2

NCODE  NDF  NFM  NRM  NC  NSE  NA  NM  NK  ND  MODE  NG  NGS  NAB
    1   10   10    2   1    4   0   0   0   0    11   1    0    1           CARD 3
              CASE 1              CASE 1, 2           CASE 1
                3                   2   1   5   1    17
              CASE 2, 3           CASE 3              CASE 2, 3

Y0, Shift Eigenvalue
1.0                                                                           CARD 4

               CASE 3 (Omit CARD 5 for CASE 1, 2)
a, ft.   c, ft.      S, sq. ft.    DM, Mach Number deviation
1.6667   2.5         4.1667        (Not used when NM=1)                       CARD 5

Mach Numbers of AIC's   CASE 3 (Omit for CASE 1, 2)
.01                                                                           CARD 6

k, Reduced Frequencies  CASE 3 (Omit for CASE 1, 2)
.05      .1      .2      .5      1.0                                          CARD 7

ρ, Density     a, Speed of Sound   CASE 3 (Omit for CASE 1, 2)
.0023769 2     1000.                                                          CARD 8
```

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PROGRAM		PUNCHING INSTRUCTIONS	GRAPHIC		PAGE OF
PROGRAMMER	DATE		PUNCH		CARD ELECTRO NUMBER*

```
C  STATEMENT NUMBER | CONT | FORTRAN STATEMENT                                                   | IDENTIFICATION SEQUENCE
    ND.Number of velocities for each altitude    CASE 3 (Omit for CASE 1, 2)
     1                                                                                           CARD 9

 K, Gain Factors                        OMIT for CASE 1, 2 3 (No gain variations)                CARD10

  Weight Matrix (Diagonal Elements)
100.            100.            100.            100.            100.            100.             CARD11
100.            100.            100.            100.

Control Surface Modes [h_δ]
0.              0.              0.              0.              0.              0.               CARD12
0.              0.              -7.5            -22.5

Structural Damping Coefficients                         CASE 2, 3 (Omit for CASE 1)
.03             .05             .08                                                              CARD13

Modal Frequencies                                       CASE 2, 3 (Omit for CASE 1)
45.             125.4           28.2                                                             CARD14

Flexible Modes [h_ε]                                    CASE 2, 3 (Omit for CASE 1)
Mode 1
1.              .411195         -.122424        -.532567        -.756204        -.756204         CARD15
-.532467        .122424         .111195         1.0
```

*A standard card form, IBM electro 888157, is available for punching statements from this form

**Number of forms per pad may vary slightly

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FORTRAN STATEMENT

```
Likewise input for Modes 2 and 3.  Each mode starts on a new card.

Rigid Body Modes: [ΦR]
Mode 1
1.         3.         1.         1.         1.
1.         1.         1.         1.                  3.                   CARD16

Likewise input for Mode 2.  Each mode starts on a new card.

X  x            X  ẋ        X  ẍ
0.          -.1667      0.          Coefficients for Control Surface 1      CARD17
     INC
       1                            Number of inputs from Servo Elements to Control Surface 1   CARD18
       K     NØ   C0          C1    Input from Servo Element 3 to Control Surface
       3      1   1.         .01    First order coefficients                CARD19
       I     NØ   C0
       1      0   1.0               Output from Servo Element 1
                                    Zero order coefficient                  CARD20
     INS
       1                            Number of Inputs to Servo Element 1     CARD21
```

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PROGRAM		PUNCHING INSTRUCTIONS	GRAPHIC		PAGE OF
PROGRAMMER	DATE		PUNCH		CARD ELECTRO NUMBER*

STATEMENT NUMBER	CONT.	FORTRAN STATEMENT	IDENTIFICATION SEQUENCE
K		NØ C0 Input from Servo Element 4 to Servo Element 1	
4		0 1.0 Zero order coefficient	
			CARD22
1		NØ C0 Output from Servo Element 3	
3		0 -1.0 Zero order coefficient	
			CARD20
INS			
2		Number of inputs to Servo Element 3	
			CARD21
K		NØ C0 Input from Servo Elements 1 and 2 to Servo Element 3	
1		0 1.0 Zero order coefficient	
2		0 -1.0 Zero order coefficient	CARD22
1		NØ C0 C1 C2 Output from Servo Element 4	
4		2 1.0 .003715 7.043 -06 Second order coefficients	
			CARD20
INS			
1		Number of inputs to Servo Element 4, Rate Gyro	
			CARD21
K		NØ C0 Input from Body Angular Rate to Rate Gyro	
0		-1 -.000833 $-K_g$, Gain Factor for Rate Gyro	
			CARD22

IBM FORTRAN Coding Form

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```
Differentiation Row Matrix for Rate Gyro.
0.          0.          1.          -27.0       27.0        -1.         CARD23
0.          0.          0.          0.

    1      NØ   C0                  Output from Servo Element 2
    2      0    1.0                 Zero order coefficient                  CARD20

  INS
    1                               Number of inputs to Servo Element 2     CARD21

    K      NØ   C0                  Input from Control Surface 1 to Servo Element 2
   -1      0    -1.0                Zero order coefficient                  CARD22

 AIC Matrices             CASE 3 (Omit for CASE 1, 2)
   MS     NZERØ
    8      1              First partition is eighth order matrix of zeroes (Omit CARD 25)   CARD24
   MS     NZERØ
    2      0              Second partition is second order non-zero matrix          CARD24

CH (1,1) Real  CH (1,1) Imag.   CH (1, 2) Real   CH (1,2) Imag.
2285.7669      -328.34515       -2300 6134       214.11571                  CARD25

CH (2,1) Real  CH (2,1) Imag.   CH (2,2) Real    CH (2,2) Imag.
.39269908      62.831853        1.1786972        -62.831853
```

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FORTRAN STATEMENT

```
Likewise, input the AIC's for the remaining reduced frequencies.

Mach Numbers for analysis   CASE 3 (Omit for CASE 1,2)
 .5                         Eigenvectors are requested.                CARD 26
```

```
                    SAMPLE PROBLEM - CASE 1 - PROGRAM MPASES

                    MISSILE AND SERVO SYSTEM - RIGID BODY IN VACUO

                         SERVO ELASTIC STABILITY ANALYSIS

                           10 DEGREES OF FREEDOM
                            6 FLEXIBLE MODES
                            2 RIGID BODY MODES
                            1 CONTROL SURFACES
                            4 SERVO ELEMENTS

                    SHIFT EIGENVALUE(GAMMA) =  1.000E+00

                           11 MODES REQUESTED

 UPPER TRIANGLE OF WEIGHT MATRIX

 ROW  1
     1.00000E+02  0.        0.        0.        0.        0.        0.        0.
     0.           0.
 ROW  2
     1.00000E+02  0.        0.        0.        0.        0.        0.        0.
     0.
 ROW  3
     1.00000E+02  0.        0.        0.        0.        0.        0.        0.
 ROW  4
     1.00000E+02  0.        0.        0.        0.        0.        0.
 ROW  5
     1.00000E+02  0.        0.        0.        0.        0.
 ROW  6
     1.00000E+02  0.        0.        0.        0.
 ROW  7
     1.00000E+02  0.        0.        0.
 ROW  8
     1.00000E+02  0.        0.
 ROW  9
     1.00000E+02  0.
 ROW 10
     1.00000E+02
```

RIGID BODY CONTROL SURFACE MODES

MODE 1

0,	0,	0,	0,	0,	0,	0,	0,
-7,50000E+00	-2,25000E+01						

RIGID BODY MODES

MODE 1

1,00000E+00	1,00000E+00	1,00000E+00	1,00000E+00	1,00000E+00	1,00000E+00	1,00000E+00	1,00000E+00
1,00000E+00	1,00000E+00						

MODE 2

-5,25000E+01	-3,75000E+01	-2,25000E+01	-7,50000E+00	7,50000E+00	2,25000E+01	3,75000E+01	5,25000E+01
6,75000E+01	6,25000E+01						

CONTROL SYSTEM DESCRIPTION

DIFFERENTIAL EQUATION FOR	VARIABLE	COEFFICIENTS 2ND ORDER	1ST ORDER	0 ORDER	GYRO/ACCEL. GAIN FACTOR
CONTROL SURF. 1	CONTROL SURF. 1	0.	-1,6670E-01	0.	
	SERVO ELEMENT 3	-0.	1,0000E-02	1,0000E+00	
SERVO ELEMENT 1	SERVO ELEMENT 1	-0.	-0.	1,0000E+00	
	SERVO ELEMENT 4	-0.	-0.	1,0000E+00	
SERVO ELEMENT 3	SERVO ELEMENT 3	-0.	-0.	-1,0000E+00	
	SERVO ELEMENT 1	-0.	-0.	1,0000E+00	
	SERVO ELEMENT 2	-0.	-0.	-1,0000E+00	
SERVO ELEMENT 4 (RATE GYRO)	SERVO ELEMENT 4	7,0430E-06	3,7150E-03	1,0000E+00	
	BODY ANGULAR RATE				-8,3300E-04
SERVO ELEMENT 2	SERVO ELEMENT 2	-0.	-0.	1,0000E+00	
	CONTROL SURF. 1	-0.	-0.	-1,0000E+00	

GYRO/ACCEL.   INPUT DIFFERENTIATION/INTERPOLATION ROW VECTOR

SERVO ELEMENT 4	0.	0.	1.0000E+00	-2.7000E+01	2.7000E+01	-1.0000E+00	0.	0.
	0.	0.						

SERVO ELEMENT	MAXIMUM ORDER	ROW ASSIGNMENT IN A AND/OR B	COLUMN ASSIGNMENT IN A AND/OR B	EIGENVECTOR ELEMENT VELOCITY	EIGENVECTOR ELEMENT DISPLACEMENT
1	0	4	10		10
2	0	5	11		11
3	1	6	9		9
4	2	7	4 AND 8	4	8

A MATRIX

3.1080097E+01	4.6621496E+02	-9.3242991E+01	0.	0.	0.	0.	0.
0.	0.	0.					
4.6621496E+02	6.4687325E+00	-7.3428856E+03	0.	0.	0.	0.	0.
0.	0.	0.					
0.	0.	0.	0.	0.	0.	-1.6670000E-01	0.
1.0000000E-02	0.	0.					
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.					
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.					
0.	0.	0.	0.	0.	0.	0.	0.
-0.	0.	0.					
0.	0.	0.	7.0430000E-06	-8.3266727E-17	-2.9980000E-01	0.	3.7150000E-03
0.	0.	0.					
0.	0.	0.	0.	1.0000000E+00	0.	0.	0.
0.	0.	0.					
0.	0.	0.	0.	0.	1.0000000E+00	0.	0.
0.	0.	0.					
0.	0.	0.	0.	0.	0.	1.0000000E+00	0.
0.	0.	0.					
0.	0.	0.	0.	0.	0.	0.	1.0000000E+00
0.	0.	0.					

B MATRIX

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.					
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.					
0.	0.	0.	0.	0.	0.	0.	0.
1.0000000E+00	0.	0.					
0.	0.	0.	0.	0.	0.	0.	1.0000000E+00
0.	1.0000000E+00	0.					
0.	0.	0.	0.	0.	0.	-1.0000000E+00	0.
0.	0.	1.0000000E+00					
0.	0.	0.	0.	0.	0.	0.	0.
-1.0000000E+00	1.0000000E+00	-1.0000000E+00					
0.	0.	0.	0.	0.	0.	0.	1.0000000E+00
0.	0.	0.					
-1.0000000E+00	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.					
0.	-1.0000000E+00	0.	0.	0.	0.	0.	0.
0.	0.	0.					
0.	0.	-1.0000000E+00	0.	0.	0.	0.	0.
0.	0.	0.					
0.	0.	0.	-1.0000000E+00	0.	0.	0.	0.
0.	0.	0.					

STABILITY ANALYSIS RESULTS

MODE	EIGENVALUE-R MU-RPS	EIGENVALUE-I OMEGA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	3.783782E-10	0.	0.	6.022076E-11		-1.831890E+09
2	-3.785559E-10	0.	0.	6.024904E-11		1.831030E+09
3	-4.839457E+00	0.	0.	7.702236E-01		1.432283E-01
4	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
5	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
6	-2.273737E-13	4.417513E-09	7.030691E-10	7.030691E-10	5.147097E-05	3.048494E+12
7	-3.882603E+02	1.236004E+02	1.967161E+01	6.485220E+01	9.528856E-01	1.785172E-03
8	-8.897610E+03	3.677455E+08	5.852852E+07	5.852852E+07	2.419502E-05	7.790262E-05

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTOR FOR MODE 1

```
 3.7837822E-10  0.              -2.7282197E-12  0.              -3.9482523E-18  0.              -8.0173592E-19  0.
 1.0000000E+00  0.              -7.2102981E-03  0.              -1.0411993E-08  0.              -8.1824774E-13  0.
-6.5674324E-19  0.               8.1824774E-13  0.               7.6442983E-11  0.
```

EIGENVECTOR FOR MODE 2

```
-3.7855585E-10  0.               2.7294997E-12  0.              -3.9519603E-18  0.              -8.0248873E-19  0.
 1.0000000E+00  0.              -7.2102957E-03  0.               1.0416881E-08  0.               8.1841305E-13  0.
-6.5723189E-19  0.              -8.1841305E-13  0.              -7.6478623E-11  0.
```

EIGENVECTOR FOR MODE 3

```
 1.0000000E+00  0.               7.8833333E-02  0.               6.8750000E-01  0.              -1.0466174E-01  0.
-2.8463474E-01  0.              -1.4636627E-02  0.              -1.4286138E-01  0.               2.1626751E-02  0.
 1.2043463E-01  0.              -2.1626751E-02  0.              -1.4286138E-01  0.
```

EIGENVECTOR FOR MODE 4

```
 1.0000000E+00  0.               7.8833333E-02  0.               6.8750000E-01  0.              -3.0159733E-35  0.
-1.0000000E-38  0.              -7.8833333E-40  0.              -6.8750000E-39  0.               3.0159733E-73  0.
-1.1468625E-37  0.              -1.2148125E-37  0.              -6.8750000E-39  0.
```

EIGENVECTOR FOR MODE 5

```
 1.0000000E+00  0.               7.8833333E-02  0.               6.8750000E-01  0.              -3.0159733E-35  0.
-1.0000000E-38  0.              -7.8833333E-40  0.              -6.8750000E-39  0.               3.0159733E-73  0.
-1.1468625E-37  0.              -1.2148125E-37  0.              -6.8750000E-39  0.
```

EIGENVECTOR FOR MODE 6

```
-2.2737485E-13  4.4175132E-09   1.7805567E-15 -3.1851582E-11   5.3815620E-16  5.5422542E-20   1.0927851E-16  1.1366746E-20
 1.0000000E+00  0.              -7.2102969E-03 -1.3836977E-08   6.2615802E-12 -1.2155857E-07   1.5394982E-14 -9.5516508E-12
 8.9515665E-17  1.0164395E-20  -1.5394982E-14  9.5516508E-12  -6.2778624E-14  8.9245978E-10
```

EIGENVECTOR FOR MODE 7

```
-6.3938589E-02  5.9369655E-03  -4.5289834E-03  4.2053506E-04  -4.3957780E-02  4.0816630E-03   1.0000000E+00  0.
 1.5393951E-04  3.3712812E-05   1.0904849E-05  2.3879908E-06   1.0583341E-04  2.3177558E-05  -2.3384937E-03 -7.4440755E-04
 2.2326603E-03  7.2122999E-04   2.3384937E-03  7.4440755E-04   1.0583341E-04  2.3177558E-05
```

EIGENVECTOR FOR MODE 8

```
 1.0000000E+00  0.               7.8833333E-02  3.1537635E-14   6.8750000E-01  2.7793978E-13  -1.8668422E-10 -8.2012510E-06
-6.5792835E-14 -2.7192717E-09  -4.6606838E-15 -1.9261588E-10  -4.5232953E-14 -1.8694993E-09  -2.2301451E-14  1.0472297E-18
-7.6544845E-13 -3.1935421E-08  -8.1099447E-13 -3.3816927E-08  -4.5525305E-14 -1.8807119E-09
```

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

```
DEFLECTIONS FOR MODE  1

  1.0000000E+00  0.          9.2154423E-01  0.           8.4308846E-01  0.           7.6463268E-01  0.
  6.6617691E-01  0.          6.0772114E-01  0.           5.2926537E-01  0.           4.5080959E-01  0.
  3.7235384E-01  0.          2.9389822E-01  0.

DEFLECTIONS FOR MODE  2

  1.0000000E+00  0.          9.2154425E-01  0.           8.4308849E-01  0.           7.6463274E-01  0.
  6.8617694E-01  0.          6.0772123E-01  0.           5.2926548E-01  0.           4.5080972E-01  0.
  3.7235391E-01  0.          2.9389805E-01  0.

DEFLECTIONS FOR MODE  3

  3.1521739E-01  0.          1.9202899E-01  0.           6.8840580E-02  0.          -5.4347826E-02  0.
 -1.7753623E-01  0.         -3.0072464E-01  0.          -4.2391304E-01  0.          -5.4710145E-01  0.
 -7.2463768E-02  0.          1.0000000E+00  0.

DEFLECTIONS FOR MODE  4

  3.1521739E-01  0.          1.9202899E-01  0.           6.8840580E-02  0.          -5.4347826E-02  0.
 -1.7753623E-01  0.         -3.0072464E-01  0.          -4.2391304E-01  0.          -5.4710145E-01  0.
 -7.2463768E-02  0.          1.0000000E+00  0.

DEFLECTIONS FOR MODE  5

  3.1521739E-01  0.          1.9202899E-01  0.           6.8840580E-02  0.          -5.4347826E-02  0.
 -1.7753623E-01  0.         -3.0072464E-01  0.          -4.2391304E-01  0.          -5.4710145E-01  0.
 -7.2463768E-02  0.          1.0000000E+00  0.

DEFLECTIONS FOR MODE  6

  1.0000000E+00  0.          9.2154424E-01 -1.0921779E-07  8.4308847E-01 -2.1843557E-07  7.6463271E-01 -3.2765336E-07
  6.6617695E-01 -4.3687115E-07  6.0772118E-01 -5.4608894E-07  5.2926542E-01 -6.5530672E-07  4.5080966E-01 -7.6452451E-07
  3.7235390E-01 -2.1239849E-07  2.9389813E-01  1.0010713E-06

DEFLECTIONS FOR MODE  7

  3.1521739E-01 -1.3754917E-15  1.9202899E-01 -1.1462431E-15  6.8840580E-02 -1.7193646E-16 -5.4347826E-02  3.4387292E-16
 -1.7753623E-01  1.1462431E-15 -3.0072464E-01  1.3754917E-15 -4.2391304E-01  2.7509833E-15 -5.4710145E-01  3.2094805E-15
 -7.2463768E-02  3.2094805E-15  1.0000000E+00  0.

DEFLECTIONS FOR MODE  8

  3.1521739E-01 -1.0830596E-09  1.9202899E-01 -7.4449654E-10  6.8840580E-02 -4.0513345E-10 -5.4347826E-02 -6.5770364E-11
 -1.7753623E-01  2.7359272E-10 -3.0072464E-01  6.1295581E-10 -4.2391304E-01  9.5231890E-10 -5.4710145E-01  1.2916820E-09
 -7.2463768E-02  9.7424236E-10  1.0000000E+00  0.
```

MINIMUM BLANK COMMON LENGTH REQUIRED =   966.
BASED ON INPUT DATA AND ANALYSES REQUESTED.

```
SAMPLE PROBLEM - CASE 2 - PROGRAM NPASES
MISSILE AND SERVO SYSTEM - FLEXIBLE BODY IN VACUO

SERVO ELASTIC STABILITY ANALYSIS

10 DEGREES OF FREEDOM
 3 FLEXIBLE MODES
 2 RIGID BODY MODES
 1 CONTROL SURFACES
 4 SERVO ELEMENTS

SHIFT EIGENVALUE(GAMMA) =  1.000E+00

17 MODES REQUESTED

UPPER TRIANGLE OF WEIGHT MATRIX

ROW  1
  1.00000E+02  0.   0.   0.   0.   0.   0.   0.
  0.           0.
ROW  2
  1.00000E+02  0.   0.   0.   0.   0.   0.   0.
  0.
ROW  3
  1.00000E+02  0.   0.   0.   0.   0.   0.   0.
ROW  4
  1.00000E+02  0.   0.   0.   0.   0.   0.
ROW  5
  1.00000E+02  0.   0.   0.   0.   0.
ROW  6
  1.00000E+02  0.   0.   0.   0.
ROW  7
  1.00000E+02  0.   0.   0.
ROW  8
  1.00000E+02  0.   0.
ROW  9
  1.00000E+02  0.
ROW 10
  1.00000E+02
```

RIGID BODY CONTROL SURFACE MODES

MODE 1

```
     0,             0,             0,            0,            0,            0,            0,            0,
-7,50000E+00   -2,25000E+01
```

FLEXIBLE MODES

MODE 1 - FREQUENCY =   45,000 CPS
STRUCTURAL DAMPING COEFFICIENT = ,030

```
 1,00000E+00    4,11195E-01   -1,22424E-01   -5,32567E-01   -7,56204E-01   -7,56204E-01   -5,32467E-01   -1,22424E-01
 4,11195E-01    1,00000E+00
```

MODE 2 - FREQUENCY =   125,400 CPS
STRUCTURAL DAMPING COEFFICIENT = ,050

```
 1,00000E+00   -1,50871E-01   -9,10842E-01   -9,90216E-01   -4,19043E-01    4,19043E-01    9,90215E-01    9,10842E-01
 1,50871E-01   -1,00000E+00
```

MODE 3 - FREQUENCY =   248,200 CPS
STRUCTURAL DAMPING COEFFICIENT = ,080

```
-8,03195E-01    6,45103E-01    9,99999E-01    1,05986E-01   -9,47894E-01   -9,47893E-01    1,05986E-01    1,00000E+00
 6,45103E-01   -8,03195E-01
```

RIGID BODY MODES

MODE 1

```
 1,00000E+00    1,00000E+00    1,00000E+00    1,00000E+00    1,00000E+00    1,00000E+00    1,00000E+00    1,00000E+00
 1,00000E+00    1,00000E+00
```

MODE 2

```
-5,25000E+01   -3,75000E+01   -2,25000E+01   -7,50000E+00    7,50000E+00    2,25000E+01    3,75000E+01    5,25000E+01
 6,75000E+01    8,25000E+01
```

CONTROL SYSTEM DESCRIPTION

DIFFERENTIAL EQUATION FOR	VARIABLE	COEFFICIENTS 2ND ORDER	1ST ORDER	0 ORDER	GYRO/ACCEL. GAIN FACTOR
CONTROL SURF. 1	CONTROL SURF. 1	0.	-1.6670E-01	0.	
	SERVO ELEMENT 3	-0.	1.0000E-02	1.0000E+00	
SERVO ELEMENT 1	SERVO ELEMENT 1	-0.	-0.	1.0000E+00	
	SERVO ELEMENT 4	-0.	-0.	1.0000E+00	
SERVO ELEMENT 3	SERVO ELEMENT 3	-0.	-0.	-1.0000E+00	
	SERVO ELEMENT 1	-0.	-0.	1.0000E+00	
	SERVO ELEMENT 2	-0.	-0.	-1.0000E+00	
SERVO ELEMENT 4 (RATE GYRO)	SERVO ELEMENT 4	7.0430E-06	3.7150E-03	1.0000E+00	
	BODY ANGULAR RATE				-8.3300E-08
SERVO ELEMENT 2	SERVO ELEMENT 2	-0.	-0.	1.0000E+00	
	CONTROL SURF. 1	-0.	-0.	-1.0000E+00	

GYRO/ACCEL.	INPUT DIFFERENTIATION/INTERPOLATION ROW VECTOR							
SERVO ELEMENT 4	0.	0.	1.0000E+00	-2.7000E+01	2.7000E+01	-1.0000E+00	0.	0.
	0.	0.						

SERVO ELEMENT	MAXIMUM ORDER	ROW ASSIGNMENT IN A AND/OR B	COLUMN ASSIGNMENT IN A AND/OR B	EIGENVECTOR ELEMENT VELOCITY	EIGENVECTOR ELEMENT DISPLACEMENT
1	0	7	16		16
2	0	8	17		17
3	1	9	15		15
4	2	10	7 AND 14	7	14

A MATRIX

1.3677063E+01	3.0942397E-04	3.5624933E-05	3.1080997E-04	1.1655374E-02	-7.9517506E+01	0.	1.0753744E+02
0.	0.	0.	0.	0.	0.	0.	0.
0.							
3.0942397E-04	1.8701554E+01	3.8040002E-06	-3.1080997E-06	-2.0979673E-04	6.6415328E+01	0.	0.
7.3675041E+02	0.	0.	0.	0.	0.	0.	0.
0.							
3.5624933E-05	3.8040002E-06	1.8468417E+01	0.	1.3986449E-04	8.1131395E+01	0.	0.
0.	3.3040999E+03	0.	0.	0.	0.	0.	0.
0.							
3.1080997E-04	-3.1080997E-06	0.	3.1080997E+01	4.6621496E+02	-9.3242991E+01	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
1.1655374E-02	-2.0979673E-04	1.3986449E-04	4.6621496E+02	6.4687325E+04	-7.3428056E+03	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.6670000E-01	0.	1.0000000E-02	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	-0.	0.	0.
0.							
0.	0.	0.	0.	0.	-0.	0.	0.
0.	0.	0.	0.	-0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	7.0030000E-06	4.5018010E-03
-1.1730458E-02	2.2080221E-02	-8.3266727E-17	-2.9980000E-01	0.	3.7150000E-03	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	1.0000000E+00
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
1.0000000E+00	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	1.0000000E+00	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	1.0000000E+00	0.	0.	0.	0.	0.
0.							

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.0000000E+00	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.0000000E+00	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.0000000E+00	0.	0.
0.							

B MATRIX

0.	0.	0.	0.	0.	0.	0.	1.0135165E+06
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
1.1610000E+07	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	4.4015161E+07	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	1.0000000E+00	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.0000000E+00	0.	1.0000000E+00
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.0000000E+00	0.	0.	0.
1.0000000E+00							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-1.0000000E+00	1.0000000E+00
-1.0000000E+00							

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.0000000E+00	0.	0.
0.							
-1.0000000E+00	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	-1.0000000E+00	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	-1.0000000E+00	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	-1.0000000E+00	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	-1.0000000E+00	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	-1.0000000E+00	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	-1.0000000E+00	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.							

STABILITY ANALYSIS RESULTS

MODE	EIGENVALUE-R MU-RPS	EIGENVALUE-I OMEGA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-4.839671E+00	0.	0.	7.702576E-01		1.432220E-01
2	-5.613621E+07	0.	0.	8.934355E+06		1.234760E-08
3	5.613801E+07	0.	0.	8.934641E+06		-1.234720E-08
4	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
5	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
6	-3.339551E-13	6.537102E-10	1.040412E-10	1.040412E-10	5.108610E-04	2.075570E+12
7	-5.968559E-13	1.554209E-08	2.473601E-09	2.473601E-09	3.840255E-05	1.161331E+12
8	-2.605417E+02	1.888656E+02	3.005889E+01	5.121531E+01	8.096506E-01	2.660408E-03
9	-1.936040E+00	3.330590E+02	5.300790E+01	5.300888E+01	5.815210E-03	3.578752E-01
10	1.349714E+02	8.356972E+02	1.330053E+02	1.347284E+02	-1.594415E-01	-5.135510E-03
11	-2.637275E+02	1.537813E+03	2.447505E+02	2.483235E+02	1.690276E-01	2.628271E-03

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTOR FOR MODE 1

1.2636392E-03	0.	-9.2141876E-05	0.	-1.4749887E-05	0.	1.0000000E+00	0.
7.0833334E-02	0.	6.8750001E-01	0.	-1.0463469E-01	0.	-2.6110022E-04	0.
1.9038872E-05	0.	3.0477044E-06	0.	-2.0662561E-01	0.	-1.4635981E-02	0.
-1.4205511E-01	0.	2.1620207E-02	0.	1.2043490E-01	0.	-2.1620207E-02	0.
-1.4205511E-01	0.						

EIGENVECTOR FOR MODE 2

1.0000000E+00	0.	-5.6622524E-01	0.	-3.5508549E-01	0.	2.3189838E-01	0.
1.6426596E-02	0.	1.5943579E-01	0.	-4.0921344E-06	0.	-1.7813815E-08	0.
1.0006632E-08	0.	6.3254273E-09	0.	-4.1309950E-09	0.	-2.9262034E-10	0.
-2.8401594E-09	0.	7.2897251E-14	0.	-5.1896096E-08	0.	-5.4038296E-08	0.
-3.0753981E-09	0.						

EIGENVECTOR FOR MODE 3

1.0000000E+00	0.	-5.6622462E-01	0.	-3.5508402E-01	0.	2.3189845E-01	0.
1.6426601E-02	0.	1.5943584E-01	0.	4.0918685E-06	0.	1.7813245E-08	0.
-1.0006297E-08	0.	-6.3251981E-09	0.	4.1308636E-09	0.	2.9261106E-10	0.
2.3400695E-09	0.	7.2890174E-14	0.	5.1094421E-08	0.	5.4036532E-08	0.
3.0753086E-09	0.						

EIGENVECTOR FOR MODE 4

1.0000000E+00	0.	-5.6622493E-01	0.	-3.5508475E-01	0.	2.3189842E-01	0.
1.6426599E-02	0.	1.5943582E-01	0.	4.7144880E-13	0.	-8.8817842E-16	0.
-1.1102230E-16	0.	0.	0.	-2.3189842E-39	0.	-1.6426599E-40	0.
-1.5943582E-39	0.	-4.7144880E-51	0.	-2.6577951E-38	0.	4.6773382E-51	0.
2.6577951E-38	0.						

EIGENVECTOR FOR MODE 5

1.0000000E+00	0.	-5.6622493E-01	0.	-3.5508475E-01	0.	2.3189842E-01	0.
1.6426599E-02	0.	1.5943582E-01	0.	4.7144880E-13	0.	-8.8817842E-16	0.
-1.1102230E-16	0.	0.	0.	-2.3189842E-39	0.	-1.6426599E-40	0.
-1.5943582E-39	0.	-4.7144880E-51	0.	-2.6577951E-38	0.	4.6773382E-51	0.
2.6577951E-38	0.						

EIGENVECTOR FOR MODE 6

9.2549531E-12	-1.8116378E-08	-6.6684805E-14	1.3053395E-10	2.5589165E-20	2.6145039E-23	-3.3395511E-13	6.5371018E-10
2.4062496E-15	-4.7101780E-12	2.4665808E-20	2.5201626E-23	2.6512523E-20	2.7088455E-23	1.7568639E-22	1.7950276E-25
-6.6603200E-26	-6.8050006E-29	-9.3429140E-30	-9.5571854E-33	1.0000000E+00	0.	-7.2053001E-03	-4.5670694E-12
-7.2158597E-16	1.4124882E-12	7.2158612E-16	-1.4124882E-12	-1.5392372E-22	-1.5682042E-25	-7.2158612E-16	1.4124882E-12
-7.2158597E-16	1.4124882E-12						

EIGENVECTOR FOR MODE 7

1.6541127E-11	-4.3872052E-07	-1.1923200E-13	3.1034710E-09	1.4464540E-17	1.1111489E-21	-5.9687036E-13	1.5542041E-08
4.3022134E-15	-1.1198543E-10	1.3942083E-17	1.0710924E-21	1.4986478E-17	1.1512442E-21	9.9308546E-20	7.6279581E-24
-3.7648147E-23	-2.8918842E-27	-5.2811787E-27	-5.5770195E-31	1.0000000E+00	0.	-7.2053001E-03	-1.0858300E-10
-1.2900588E-15	3.3582191E-11	1.2901458E-15	-3.3582191E-11	-8.7006976E-20	-6.7663296E-25	-1.2901458E-15	3.3582191E-11
-1.2900588E-15	3.3582191E-11						

EIGENVECTOR FOR MODE 8

-1.4195874E-01	1.3521966E-01	5.6706265E-03	-2.4401306E-02	4.9563911E-04	-4.1625411E-03	-6.2647192E-02	1.4573352E-02
-4.4375745E-03	1.0323412E-03	-4.3070743E-02	1.0019942E-02	1.0000000E+00	0.	6.0379711E-04	-8.1304370E-05
-5.8772309E-05	5.1052247E-05	-8.8389764E-06	9.5691501E-06	1.8420288E-04	7.7593060E-05	1.3047981E-05	5.4961377E-06
1.2664208E-04	5.3344769E-05	-2.5160426E-03	-1.8238684E-03	2.3893997E-03	1.7705237E-03	2.5160426E-03	1.8238684E-03
1.2664208E-04	5.3344769E-05						

EIGENVECTOR FOR MODE 9

1.0000000E+00	0.	3.4372826E-02	-2.5624397E-03	4.7371822E-03	-3.1702739E-04	6.4751727E-02	-3.9623410E-03
4.5870389E-03	-2.8066582E-04	4.4522436E-02	-2.7241093E-03	-7.3888112E-01	-1.6062235E-01	-1.7459707E-05	-3.0023699E-03
-8.1133890E-06	-1.8315625E-04	-1.0345433E-06	-1.4217238E-05	-1.3026960E-05	-1.9433945E-04	-9.2275096E-07	-1.3767087E-05
-8.9561326E-06	-1.3362526E-04	-4.6934705E-04	2.2211988E-03	4.7830319E-04	-2.0875736E-03	4.6934705E-04	-2.2211989E-03
-8.9561326E-06	-1.3362526E-04						

EIGENVECTOR FOR MODE 10

-4.0477406E-01	2.3150711E-02	3.3080496E-01	4.5016511E-01	-4.2091772E-02	2.9544428E-02	-8.3834605E-02	9.0366118E-03
-5.9384783E-03	6.4010324E-04	-5.7638580E-02	6.2127823E-03	1.0000000E+00	0.	-4.9240314E-05	4.7640226E-04
5.8728271E-04	-3.0099248E-04	2.0526404E-05	5.4651470E-05	-5.2517023E-06	9.9468778E-05	-3.7201901E-07	7.0459236E-06
-3.6108462E-06	6.8387475E-05	1.8834792E-04	-1.1661861E-03	-1.8473707E-04	1.0977986E-03	-1.8834792E-04	1.1661861E-03
-3.6108462E-06	6.8387475E-05						

EIGENVECTOR FOR MODE 11

-3.6283958E-01	2.3971351E-02	2.6052818E-01	-3.6601856E-02	5.4863812E-02	-4.8153300E-01	-8.1490000E-02	4.9114054E-03
-5.7723753E-03	3.4790465E-04	-5.0026428E-02	3.3767395E-03	1.0000000E+00	0.	5.4450016E-05	2.2660732E-04
-5.1344975E-05	-1.6060935E-04	-3.1012587E-04	1.7508568E-05	1.1930551E-05	5.0944817E-05	8.4510727E-07	3.6086952E-06
8.2025699E-06	3.5025842E-05	-1.0833277E-04	-6.3169570E-04	1.0013020E-04	5.9666986E-04	1.0833277E-04	6.3169570E-04
8.2025699E-06	3.5025842E-05						

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 1

3.1513816E-01	0.	1.9199868E-01	0.	6.8861415E-02	0.	-5.4288805E-02	0.
-1.7745960E-01	0.	-3.8065866E-01	0.	-4.2389148E-01	0.	-5.4715803E-01	0.
-7.2532795E-02	0.	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 2

-3.1217088E-01	0.	4.1197519E-01	0.	3.5161155E-01	0.	-3.5060654E-01	0.
-6.1066111E-01	0.	1.9582477E-01	0.	1.0000000E+00	0.	-3.5718269E-01	0.
-8.5413544E-01	0.	5.2594535E-01	0.				

DEFLECTIONS FOR MODE 3

-3.1276419E-01	0.	4.1197415E-01	0.	3.5160985E-01	0.	-3.5060622E-01	0.
-6.1065775E-01	0.	1.9582893E-01	0.	1.0000000E+00	0.	-3.5719249E-01	0.
-8.5414272E-01	0.	5.2595516E-01	0.				

DEFLECTIONS FOR MODE 4

1.0000000E+00	0.	3.4874322E-01	0.	-2.1002680E-01	0.	-5.8341689E-01	0.
-7.1874167E-01	0.	-6.2562100E-01	0.	-3.6328011E-01	0.	-7.6166667E-03	0.
3.8227011E-01	0.	7.7777778E-01	0.				

DEFLECTIONS FOR MODE 5

1.0000000E+00	0.	3.4874322E-01	0.	-2.1002680E-01	0.	-5.8341689E-01	0.
-7.1874167E-01	0.	-6.2562100E-01	0.	-3.6328011E-01	0.	-7.6166667E-03	0.
3.8227011E-01	0.	7.7777778E-01	0.				

DEFLECTIONS FOR MODE 6

1.0000000E+00	0.	4.2158369E-01	-3.6062437E-11	8.4316737E-01	-7.2124873E-11	7.6475186E-01	-1.0818731E-10
6.8633474E-01	-1.4424975E-10	6.0791843E-01	-1.8031218E-10	5.2950211E-01	-2.1637462E-10	4.5108580E-01	-2.5243786E-10
3.7266948E-01	-2.9618565E-10	2.9425317E-01	-3.4762040E-10				

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DEFLECTIONS FOR MODE  7

  1.0000000E+00  0.              9.2158369E-01 -8.5739169E-10    8.4316737E-01 -1.7147834E-09    7.6475106E-01 -2.5721751E-09
  6.8633474E-01 -3.4295668E-09   6.0791843E-01 -4.2869585E-09    5.2950211E-01 -5.1443501E-09    4.5108580E-01 -6.0017418E-09
  3.7266948E-01 -7.0418734E-09   2.9425317E-01 -8.2647449E-09

DEFLECTIONS FOR MODE  8

  1.0286034E-01  1.8022370E-01   1.2109424E-01  7.1235139E-02    1.2436215E-01 -3.0023243E-02    9.2804327E-02 -1.1449656E-01
  5.8835712E-03 -1.6819580E-01  -1.5145339E-01 -1.6591030E-01   -3.8376126E-01 -7.3970086E-02   -6.8238845E-01  1.3064094E-01
 -2.2940151E-01  1.7049621E-01   1.0000000E+00  0.

DEFLECTIONS FOR MODE  9

  1.0000000E+00  0.              3.5323579E-01  4.3585593E-03   -2.1929779E-01  7.2374615E-03   -6.2690844E-01  6.9412312E-03
 -7.9181605E-01  2.3698823E-03  -6.7686704E-01 -6.1501232E-03   -3.0580022E-01 -1.6842565E-02    2.5624155E-01 -2.7475785E-02
  5.3811477E-01 -8.9477464E-03   4.7229743E-01  3.8509085E-02

DEFLECTIONS FOR MODE 10

 -7.8829154E-01  3.6698691E-01   9.4574661E-02 -2.0175182E-01    7.1539095E-01 -4.9052319E-01    8.3927476E-01 -3.2848402E-01
  4.2266022E-01  1.3233370E-01  -2.9418074E-01  5.1720405E-01   -8.6911245E-01  4.7410691E-01   -9.1188355E-01 -1.2743405E-01
 -2.0783231E-01 -3.6248849E-01   1.0000000E+00  0.

DEFLECTIONS FOR MODE 11

 -7.3720253E-01  8.0522790E-02   6.1118665E-01  2.2588085E-02    9.2680558E-01 -9.1345116E-02    6.9445662E-02 -1.8962641E-01
 -9.2142989E-01 -1.1274572E-01  -8.9602726E-01  1.5609706E-01    1.3352506E-01  3.3447848E-01    1.0000000E+00  0.
  6.3169984E-01 -1.9233059E-03  -8.1800310E-01 -7.6385779E-03


MINIMUM BLANK COMMON LENGTH REQUIRED =  2106.
BASED ON INPUT DATA AND ANALYSES REQUESTED.
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                    SAMPLE PROBLEM - CASE 3 - PROGRAM MPASES

        MISSILE AND SERVO SYSTEM - FLEX, BODY IN INCOMPRESS, AIRSTREAM AT S, L,

                    AERO-SERVO-ELASTIC STABILITY ANALYSIS

                         10 DEGREES OF FREEDOM
                          1 FLEXIBLE MODES
                          2 RIGID BODY MODES
                          1 CONTROL SURFACES
                          4 SERVO ELEMENTS

                    SHIFT EIGENVALUE(GAMMA) = 1,000E+00

                         17 MODES REQUESTED

                          1 ALTITUDE VARIATIONS

              AIC MATRICES FOR 1 MACH NUMBERS AND  5 REDUCED FREQUENCIES

                    REFERENCE SEMI-CHORD = 1,25000E+00 FT
                               SEMI-SPAN = 1,66670E+00 FT
                            SURFACE AREA = 8,16670E+00 SQ FT

                    MACH NUMBER DEVIATION =-0,

 UPPER TRIANGLE OF WEIGHT MATRIX

 ROW  1
    1,00000E+02   0,          0,          0,          0,          0,          0,          0,
    0,            0,
 ROW  2
    1,00000E+02   0,          0,          0,          0,          0,          0,          0,
    0,
 ROW  3
    1,00000E+02   0,          0,          0,          0,          0,          0,          0,
 ROW  4
    1,00000E+02   0,          0,          0,          0,          0,          0,
 ROW  5
    1,00000E+02   0,          0,          0,          0,          0,
 ROW  6
    1,00000E+02   0,          0,          0,          0,
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```
ROW 7
  1.00000E+02   0.          0.          0.
ROW 8
  1.00000E+02   0.          0.
ROW 9
  1.00000E+02   0.
ROW 10
  1.00000E+02

RIGID BODY CONTROL SURFACE MODES

MODE 1
  0.            0.
 -7.50000E+00  -2.25000E+01   0.          0.          0.          0.          0.          0.

FLEXIBLE MODES

MODE 1 - FREQUENCY =      85.000 CPS
         STRUCTURAL DAMPING COEFFICIENT = .030

  1.00000E+00   4.11195E-01  -1.22424E-01  -5.32567E-01  -7.56208E-01  -7.56208E-01  -5.32467E-01  -1.22424E-01
  4.11195E-01   1.00000E+00
MODE 2 - FREQUENCY =     125.400 CPS
         STRUCTURAL DAMPING COEFFICIENT = .050

  1.00000E+00  -1.50871E-01  -9.10642E-01  -9.90216E-01  -4.19043E-01   4.19043E-01   9.90215E-01   9.10842E-01
  1.50871E-01  -1.00000E+00
MODE 3 - FREQUENCY =     248.200 CPS
         STRUCTURAL DAMPING COEFFICIENT = .080

 -8.03195E-01   6.45103E-01   9.99999E-01   1.05986E-01  -9.47894E-01  -9.47893E-01   1.05986E-01   1.00000E+00
  6.45103E-01  -8.03195E-01
```

RIGID BODY MODES

MODE 1

1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00
1.00000E+00	1.00000E+00						

MODE 2

-5.25000E+01	-3.75000E+01	-2.25000E+01	-7.50000E+00	7.50000E+00	2.25000E+01	3.75000E+01	5.25000E+01
6.75000E+01	8.25000E+01						

CONTROL SYSTEM DESCRIPTION

DIFFERENTIAL EQUATION FOR	VARIABLE	COEFFICIENTS 2ND ORDER	1ST ORDER	0 ORDER	GYRO/ACCEL. GAIN FACTOR
CONTROL SURF. 1	CONTROL SURF. 1	0.	-1.6670E-01	0.	
	SERVO ELEMENT 3	-0.	1.0000E-02	1.0000E+00	
SERVO ELEMENT 1	SERVO ELEMENT 1	-0.	-0.	1.0000E+00	
	SERVO ELEMENT 4	-0.	-0.	1.0000E+00	
SERVO ELEMENT 3	SERVO ELEMENT 3	-0.	-0.	-1.0000E+00	
	SERVO ELEMENT 1	-0.	-0.	1.0000E+00	
	SERVO ELEMENT 2	-0.	-0.	-1.0000E+00	
SERVO ELEMENT 4 (RATE GYRO)	SERVO ELEMENT 4	7.0430E-06	3.7150E-03	1.0000E+00	
	BODY ANGULAR RATE				-8.3300E-04
SERVO ELEMENT 2	SERVO ELEMENT 2	-0.	-0.	1.0000E+00	
	CONTROL SURF. 1	-0.	-0.	-1.0000E+00	

GYRO/ACCEL. INPUT DIFFERENTIATION/INTERPOLATION ROW VECTOR

SERVO ELEMENT 4	0.	0.	1.0000E+00	-2.7000E+01	2.7000E+01	-1.0000E+00	0.	0.
	0.	0.						

SERVO ELEMENT	MAXIMUM ORDER	ROW ASSIGNMENT IN A AND/OR B	COLUMN ASSIGNMENT IN A AND/OR B	EIGENVECTOR ELEMENT VELOCITY	DISPLACEMENT
1	0	7	16		16
2	0	8	17		17
3	1	9	15		15
4	2	10	7 AND 14	7	14

AIC MATRIX FOR MACH NUMBER = 1.0000E-02 AND REDUCED FREQUENCY = 5.000000E-02

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
2.28577E+03	-3.28345E+02	-2.30061E+03	2.14116E+02				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
3.92699E-01	6.28319E+01	1.17810E+00	-6.28319E+01				

AIC MATRIX FOR MACH NUMBER = 1.0000E-02 AND REDUCED FREQUENCY = 1.000000E-01

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
5.23891E+02	-1.08261E+02	-5.33147E+02	5.59893E+01				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
3.92699E-01	3.14159E+01	1.17810E+00	-3.14159E+01				

AIC MATRIX FOR MACH NUMBER = 1.0000E-02 AND REDUCED FREQUENCY = 2.000000E-01

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
1.15466E+02	-2.96290E+01	-1.19821E+02	6.77142E+00				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
3.92699E-01	1.57080E+01	1.17810E+00	-1.57080E+01				

AIC MATRIX FOR MACH NUMBER = 1.0000E-02 AND REDUCED FREQUENCY = 5.000000E-01

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
1.62059E+01	-3.78774E+00	-1.65289E+01	-3.72614E+00				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
3.92699E-01	6.28319E+00	1.17810E+00	-6.28319E+00				

AIC MATRIX FOR MACH NUMBER = 1.0000E-02 AND REDUCED FREQUENCY = 1.000000E+00

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
4.56747E+00	-6.30033E-01	-3.62670E+00	-2.75934E+00				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
3.92699E-01	3.14159E+00	1.17810E+00	-3.14159E+00				

A MATRIX

1.2677063E+01	3.0942397E-04	3.5424933E-05	3.1080997E-04	1.1655374E-02	-7.9517506E+01	0.	1.0688701E+02
7.0113483E+00	1.1560743E+01	6.0130891E+00	3.2632445E+02	3.4460890E+01	0.	0.	0.
0.							
3.0942397E-04	1.8701554E+01	3.8040002E-06	-3.1080997E-06	-2.0979673E-04	6.6415328E+01	0.	-6.4992048E+00
7.5156774E+02	1.9640977E+01	2.2062544E+00	-3.9758435E+01	1.7213370E+02	0.	0.	0.
0.							
3.5424933E-05	3.8040002E-06	1.8468417E+01	0.	1.3986449E-04	4.1131395E+01	0.	-1.1884591E+01
3.2234704E+01	2.3485598E+03	9.4336308E+00	2.3518586E+02	3.3083199E+02	0.	0.	0.
0.							
3.1080997E-04	-3.1080997E-06	0.	3.1080997E+01	4.6621496E+02	-9.3242991E+01	0.	-8.1401976E+00
2.9670094E+01	4.4246815E+01	1.4623449E+01	6.2652319E+02	2.5088374E+02	0.	0.	0.
0.							
1.1655374E-02	-2.0979673E-04	1.3986449E-04	4.6621496E+02	6.4687325E+04	-7.3428856E+03	0.	-4.8076209E+02
1.8819487E+03	2.8176739E+03	9.8768280E+02	4.4040502E+04	1.5184466E+04	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.6670000E-01	0.	1.0000000E-02	0.
0.							
0.	0.	0.	0.	0.	0.	-0.	0.
0.	0.	0.	0.	0.	-0.	0.	0.
0.							
0.	0.	0.	0.	0.	-0.	0.	0.
0.	0.	0.	0.	-0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-0.	0.
0.							
0.	0.	0.	0.	0.	0.	7.0430000E-06	4.5018810E-03
-1.1738450E-02	2.2080221E-02	-8.3266727E-17	-2.9988000E-01	0.	3.7150000E-03	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	1.0000000E+00
0.	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
1.0000000E+00	0.	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	1.0000000E+00	0.	0.	0.	0.	0.	0.
0.							
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	1.0000000E+00	0.	0.	0.	0.	0.
0.							

```
0.              0.              0.              0.              0.              0.              0.              0.
0.              0.              0.              1.0000000E+00   0.              0.              0.              0.
0.

0.              0.              0.              0.              0.              0.              0.              0.
0.              0.              0.              0.              -1.0000000E+00  0.              0.              0.
0.

0.              0.              0.              0.              0.              0.              0.              0.
0.              0.              0.              0.              0.              1.0000000E+00   0.              0.
0.
```

B MATRIX

```
0.              0.              0.              0.              0.              0.              0.              1.0149389E+06
-2.7743178E+03  -3.4884856E+03  6.1877868E+00   3.6589210E+04   -3.6217943E+04  0.              0.              0.
0.

0.              0.              0.              0.              0.              0.              0.              5.2191991E+02
1.1608990E+07   -1.2799687E+03  2.3002362E+00   1.3427251E+04   -1.3289237E+04  0.              0.              0.
0.

0.              0.              0.              0.              0.              0.              0.              2.2315908E+03
-4.3525235E+03  4.4909688E+07   9.7595635E+00   5.7407041E+04   -5.6821467E+04  0.              0.              0.
0.

0.              0.              0.              0.              0.              0.              0.              3.4592364E+03
-6.7469861E+03  -8.4837882E+03  1.5079608E+01   8.8985089E+04   -8.8080312E+04  0.              0.              0.
0.

0.              0.              0.              0.              0.              0.              0.              2.3349818E+05
-4.5542133E+05  -5.7265556E+05  1.0175456E+03   6.0064677E+06   -5.9454149E+06  0.              0.              0.
0.

0.              0.              0.              0.              0.              0.              0.              0.
0.              0.              0.              0.              0.              0.              1.0000000E+00   0.
0.

0.              0.              0.              0.              0.              0.              0.              0.
0.              0.              0.              0.              0.              1.0000000E+00   0.              1.0000000E+00
0.

0.              0.              0.              0.              0.              0.              0.              0.
0.              0.              0.              0.              -1.0000000E+00  0.              0.              0.
-1.0000000E+00

0.              0.              0.              0.              0.              0.              0.              0.
0.              0.              0.              0.              0.              0.              -1.0000000E+00  1.0000000E+00
-1.0000000E+00

0.              0.              0.              0.              0.              0.              0.              0.
0.              0.              0.              0.              0.              1.0000000E+00   0.              0.
0.
```

-1.0000000E+00 0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.
0. 0. 0.	-1.0000000E+00 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.
0. 0. 0.	0. 0.	-1.0000000E+00 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.
0. 0. 0.	0. 0.	0. 0.	-1.0000000E+00 0.	0. 0.	0. 0.	0. 0.	0. 0.
0. 0. 0.	0. 0.	0. 0.	0. 0.	-1.0000000E+00 0.	0. 0.	0. 0.	0. 0.
0. 0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	-1.0000000E+00 0.	0. 0.	0. 0.
0. 0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	0. 0.	-1.0000000E+00 0.	0. 0.

STABILITY ANALYSIS RESULTS

SPEED OF SOUND = 1.00000E+03 FT/SEC

DENSITY = 2.37692E-03 SLUGS/CU FT

MACH NUMBER = 5.00000E-01

VELOCITY = 5.00000E+02 FT/SEC, 2.96043E+02 KNOTS

MODE	EIGENVALUE-R MU-RPS	EIGENVALUE-I OMEGA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	REDUCED FREQUENCY	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-2.165143E+00	0.	0.	3.445931E-01	0.		3.201393E-01
2	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
3	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
4	-8.930284E-05	1.405643E-02	2.237150E-03	2.237195E-03	3.513954E-05	6.352982E-03	7.761829E+03
5	-1.862752E+00	1.330917E+01	2.116221E+00	2.139311E+00	3.346149E-02	1.400681E-01	3.681564E-01
6	-2.605184E+02	1.888337E+02	3.005361E+01	5.120932E+01	4.697925E-01	8.096727E-01	2.660646E-03
7	-1.964972E+00	3.329844E+02	5.295611E+01	5.299703E+01	8.325686E-01	5.900991E-03	3.527517E-01
8	1.348340E+02	8.354246E+02	1.329619E+02	1.346826E+02	2.080712E+00	-1.593339E-01	-5.140745E-03
9	-2.639520E+02	1.537779E+03	2.447452E+02	2.483243E+02	3.844531E+00	1.691710E-01	2.626035E-03

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTOR FOR MODE 1

```
 1.1392527E-04  0.              1.6629160E-06  0.              3.1421304E-06  0.              1.0000000E+00  0.
 5.0747546E-02  0.              5.2630719E-02  0.             -3.3214361E-02  0.             -5.2617987E-05  0.
-7.6803998E-07  0.             -1.4512349E-06  0.             -4.6186336E-01  0.             -2.3438432E-02  0.
-2.4308201E-02  0.              1.5340496E-02  0.              8.9677045E-03  0.             -1.5340496E-02  0.
-2.4308201E-02  0.
```

EIGENVECTOR FOR MODE 2

```
 1.0000000E+00  0.             -5.6622493E-01  0.             -3.5508475E-01  0.              2.3189842E-01  0.
 1.6426599E-02  0.              1.5943582E-01  0.              1.3986315E-11  0.             -6.6613381E-16  0.
-2.5873828E-16  0.              1.1537001E-17  0.             -2.3189842E-39  0.              4.4100090E-16  0.
-1.5943582E-39  0.             -1.3986315E-49  0.             -2.6577951E-38  0.              1.3981900E-49  0.
 2.6577951E-38  0.
```

EIGENVECTOR FOR MODE 3

```
 1.0000000E+00  0.             -5.6622493E-01  0.             -3.5508475E-01  0.              2.3189842E-01  0.
 1.6426599E-02  0.              1.5943582E-01  0.              1.3986315E-11  0.             -6.6613381E-16  0.
-2.5873828E-16  0.              1.1537001E-17  0.             -2.3189842E-39  0.              4.4100090E-16  0.
-1.5943582E-39  0.             -1.3986315E-49  0.             -2.6577951E-38  0.              1.3981900E-49  0.
 2.6577951E-38  0.
```

EIGENVECTOR FOR MODE 4

```
 1.5466159E-11  1.3234147E-10  5.9191474E-13 -3.1165894E-11  2.1016862E-13 -1.1011013E-11 -8.9302044E-05  1.4056429E-02
 3.7414667E-08 -2.3806637E-06 -1.0035549E-08 -1.9743494E-10  1.0034940E-08  2.2094750E-10  1.0044404E-08  1.2807895E-10
-2.2174824E-09 -2.8241862E-11 -7.8340785E-10 -9.9742656E-12  1.0000000E+00  0.             -1.6938906E-04 -1.5856003E-06
-9.5097136E-09  7.1400767E-07  1.1102645E-04 -7.1397500E-07 -1.6729321E-09 -3.2677281E-11 -1.1102645E-08  7.1397500E-07
-9.5097136E-09  7.1400767E-07
```

EIGENVECTOR FOR MODE 5

```
-3.2820638E-03 -7.5761605E-04 -1.9610102E-04 -8.4071092E-05 -1.5777230E-04 -5.3625593E-05  1.0000000E+00  0.
 4.0870363E-02 -6.7296406E-04 -6.8141720E-02 -3.6197312E-02 -1.4651626E-02  1.0993210E-01 -2.1613479E-05  2.4971920E-04
-4.1277164E-06  1.5474474E-05 -2.3061151E-06  1.2180630E-05 -1.0420429E-02 -7.3062033E-02 -5.3798121E-04 -3.4455536E-03
-1.9563016E-03  5.3966496E-03  1.4143461E-02 -8.9990547E-04 -1.2187159E-02 -4.4967441E-03 -1.4143461E-02  8.9990547E-04
-1.9563016E-03  5.3960490E-03
```

EIGENVECTOR FOR MODE 6

```
-1.4150340E-01  1.3504009E-01  5.5540719E-03 -2.4348810E-02  4.5303093E-04 -4.1494879E-03 -6.2586589E-02  1.4404512E-02
-4.4300874E-03  1.0274177E-03 -4.3068764E-02  1.0020632E-02  1.0000000E+00  0.              6.0239233E-04 -8.1710829E-05
-5.8390211E-05  5.1134405E-05 -8.7086195E-06  9.6154713E-06  1.6376687E-04  7.7409504E-05  1.3021690E-05  5.4940745E-06
 1.2665597E-04  5.3340874E-05 -2.5164054E-03 -1.8239865E-03  2.3897494E-03  1.7706457E-03  2.5164054E-03  1.8239805E-03
 1.2665597E-04  5.3340874E-05
```

EIGENVECTOR FOR MODE 7

1.0000000E+00	0.	3.4332129E-02	-2.5507323E-03	4.7362692E-03	-3.4186426E-04	6.4642836E-02	-3.7956784E-03
4.5805491E-03	-2.7392028E-04	4.4560388E-02	-2.7473742E-03	-7.3960005E-01	-1.6046628E-01	-1.7721216E-05	-3.0030393E-03
-8.2683564E-06	-1.0305553E-04	-1.1105643E-05	-1.4217144E-05	-1.2544121E-05	-1.9405772E-04	-9.0376662E-07	-1.3750775E-05
-9.6401369E-06	-1.3376791E-04	-4.6077993E-04	2.2230917E-03	4.7782007E-04	-2.0901238E-03	4.6077993E-04	-2.2230917E-03
-9.6401369E-06	-1.3376791E-04						

EIGENVECTOR FOR MODE 8

-4.0417764E-01	2.3588187E-02	3.3080152E-01	4.5012142E-01	-4.2014120E-02	2.9601831E-02	-8.3648049E-02	9.0833510E-03
-5.9271008E-03	6.4479933E-04	-5.7638277E-02	6.2150324E-03	1.0000000E+00	0.	-4.8542644E-05	4.7595805E-04
5.8740025E-04	-3.0116423E-04	2.6623073E-05	5.4587591E-05	-5.1530110E-06	9.9294719E-05	-3.6375953E-07	7.0360077E-06
-3.6019497E-06	6.8411457E-05	1.6828554E-04	-1.1666077E-03	-1.8468359E-04	1.0961962E-03	-1.6828554E-04	1.1666077E-03
-3.6019497E-06	6.8411457E-05						

EIGENVECTOR FOR MODE 9

-3.6313365E-01	2.4065906E-02	2.6076356E-01	-3.6669237E-02	5.5066237E-02	-4.8155314E-01	-8.1544694E-02	4.9438930E-03
-5.7766779E-03	3.4975299E-04	-5.6025938E-02	3.3766478E-03	1.0000000E+00	0.	5.4574415E-05	2.2677417E-04
-5.1436254E-05	-1.6074274E-04	-3.1015700E-04	1.7427940E-05	1.1964375E-05	5.0973943E-05	8.4726351E-07	3.6110780E-06
8.2075271E-06	3.5024236E-05	-1.0842430E-04	-6.3167796E-04	1.0021678E-04	5.9665366E-04	1.0842430E-04	6.3167790E-04
8.2075271E-06	3.5024236E-05						

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 1

-4.1285654E-01	0.	-2.2402234E-01	0.	-3.5187275E-02	0.	1.5365059E-01	0.
3.4249387E-01	0.	5.3134440E-01	0.	7.2020197E-01	0.	9.0906440E-01	0.
1.0000000E+00	0.	9.9380706E-01	0.				

DEFLECTIONS FOR MODE 2

-6.6970874E-01	0.	-4.6613487E-01	0.	-2.6674599E-01	0.	-7.4940858E-02	0.
1.0867864E-01	0.	2.8657399E-01	0.	4.6258369E-01	0.	6.3977149E-01	0.
8.1916611E-01	0.	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 3

-6.6970874E-01	0.	-4.6613487E-01	0.	-2.6674599E-01	0.	-7.4940858E-02	0.
1.0867864E-01	0.	2.8657399E-01	0.	4.6258369E-01	0.	6.3977149E-01	0.
8.1916611E-01	0.	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 4

1.0000000E+00	0.	9.9748155E-01	-2.3366618E-05	9.9496311E-01	-4.6733228E-05	9.9244466E-01	-7.0099830E-05
9.8992622E-01	-9.3466425E-05	9.8740778E-01	-1.1683301E-04	9.8488933E-01	-1.4019957E-04	9.8237090E-01	-1.6356608E-04
9.7985253E-01	-1.9224042E-04	9.7733424E-01	-2.2622258E-04				

```
DEFLECTIONS FOR MODE  5

   2506131E-01  3.2076322E-02  -1.1655890E-01  1.7709453E-02  -8.0700742E-03  3.3416832E-03   1.0036432E-01 -1.1032736E-02
  .0868491E-01 -2.5422970E-02   3.1685401E-01 -3.9834080E-02   4.2488867E-01 -5.4261087E-02   5.3285139E-01 -6.8691747E-02
 7.2454773E-01 -5.0604832E-02   1.0000000E+00  0.

DEFLECTIONS FOR MODE  6

 1.0220969E-01  1.7981085E-01   1.2043849E-01  7.1180922E-02   1.2379274E-01 -2.9792633E-02   9.2460324E-02 -1.1412758E-01
 5.8955712E-03 -1.6791839E-01  -1.5094984E-01 -1.6609437E-01  -3.8254593E-01 -7.5180622E-02  -6.8013156E-01  1.2769964E-01
-2.2750410E-01  1.6786364E-01   1.0000000E+00  0.

DEFLECTIONS FOR MODE  7

 1.0000000E+00  0.              3.5331856E-01  4.3347907E-03  -2.1916561E-01  7.2190099E-03  -6.2679355E-01  6.9689362E-03
-7.9100224E-01  2.4748887E-03  -6.7702902E-01 -5.9416503E-03  -3.0618271E-01 -1.6480374E-02   2.5562791E-01 -2.6870158E-02
 5.3702786E-01 -7.8946874E-03   4.7049592E-01  4.0175678E-02

DEFLECTIONS FOR MODE  8

-7.8843667E-01  3.9029301E-01   9.3889727E-02 -2.0222521E-01   7.1485628E-01 -4.9362029E-01   8.3971778E-01 -3.3263221E-01
 4.2359733E-01  1.3072244E-01  -2.9247444E-01  5.1869568E-01  -8.6870930E-01  4.7811844E-01  -9.1400002E-01 -1.2319309E-01
-2.1023545E-01 -3.5993278E-01   1.0000000E+00  0.

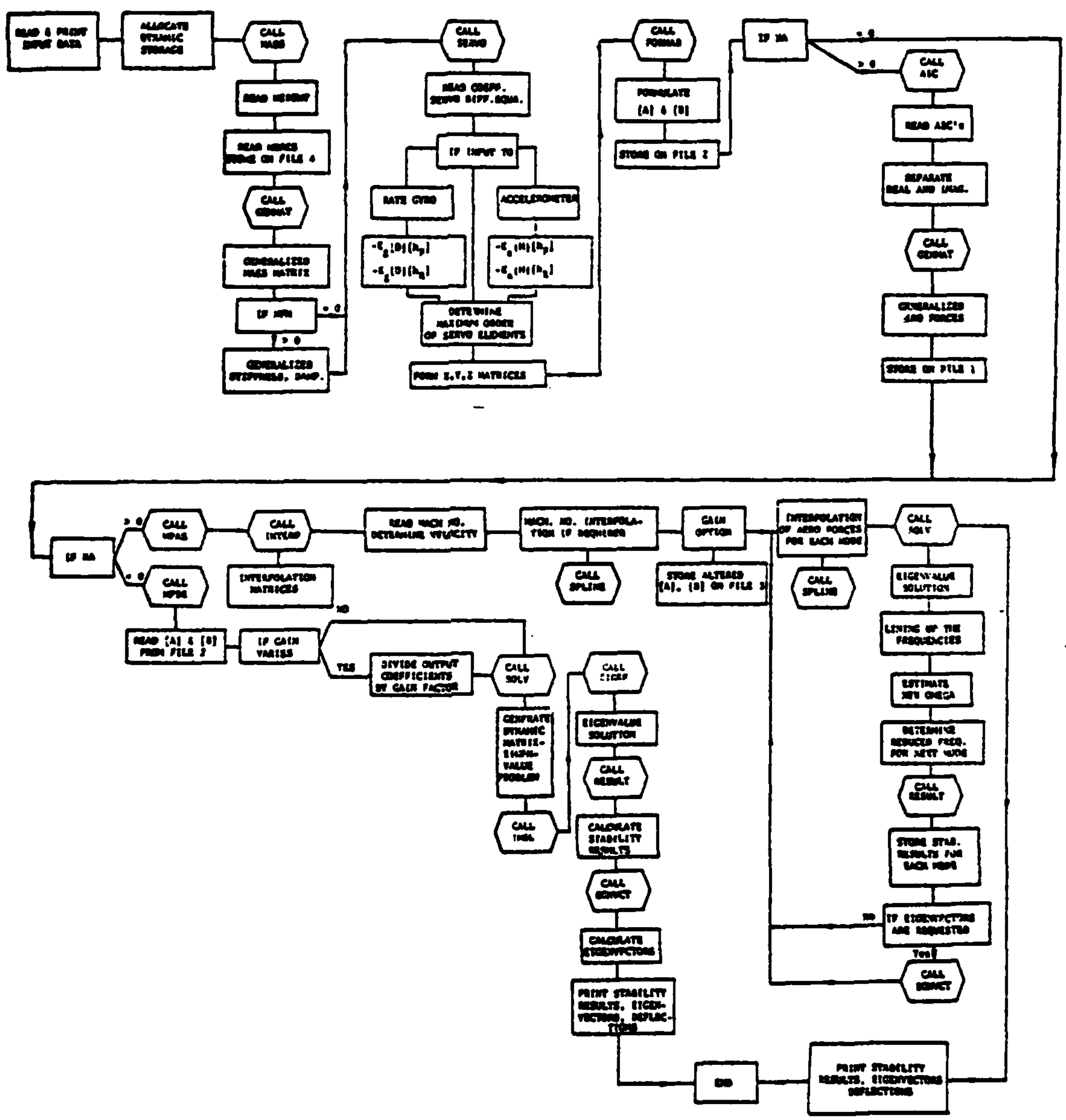
DEFLECTIONS FOR MODE  9

-7.3720154E-01  8.0684275E-02   6.1124518E-01  2.2545943E-02   9.2681927E-01 -9.1539531E-02   6.9330128E-02 -1.0983151E-01
-9.2158037E-01 -1.1276127E-01  -8.9605228E-01  1.5637295E-01   1.3364941E-01  3.3486304E-01   1.0000000E+00  0.
 6.3141288E-01 -1.9320635E-01  -8.1860877E-01 -9.6625770E-03
```

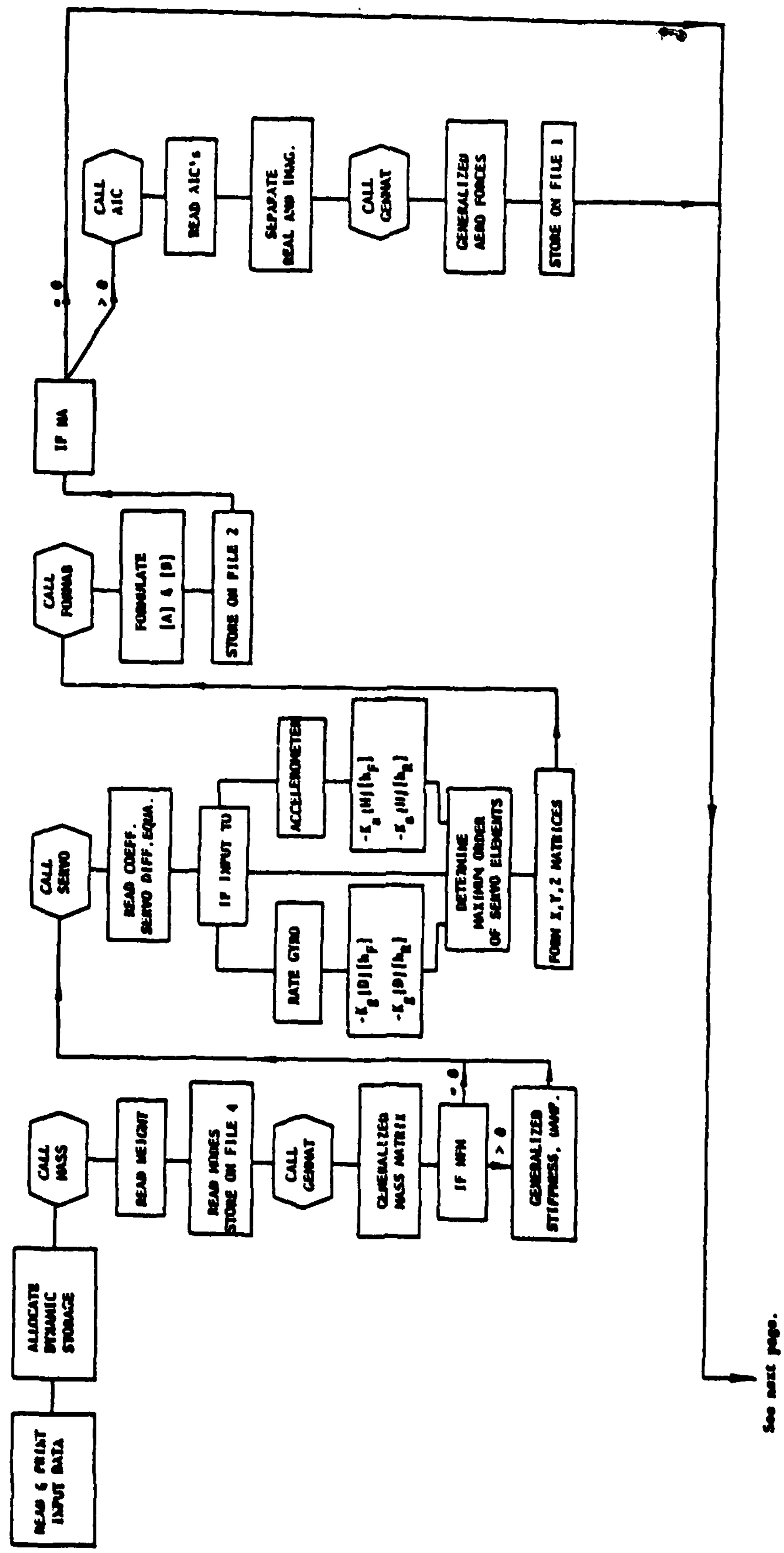
MINIMUM BLANK COMMON LENGTH REQUIRED = 2646.
BASED ON INPUT DATA AND ANALYSES REQUESTED.

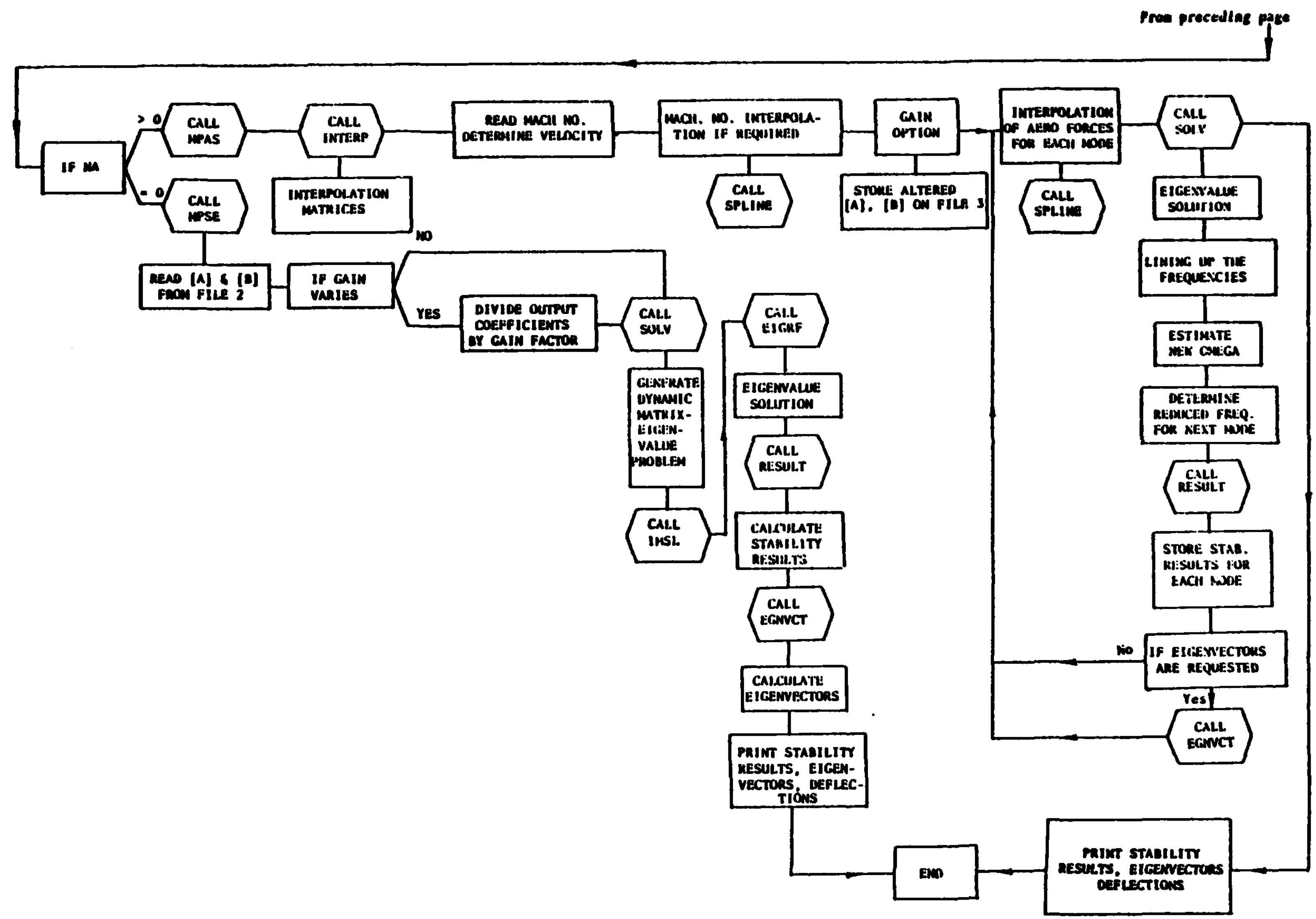
SECTION V

PROGRAM FLOW CHART



Note: See next two pages for larger scale.


READ & PRINT INPUT DATA
ALLOCATE DYNAMIC STORAGE
CALL MASS
READ WEIGHT
READ MODES STORE ON FILE 4
CALL GENMAT
GENERALIZED MASS MATRIX
IF NFN
= 0
> 0
GENERALIZED STIFFNESS, DAMP.
CALL SERVO
READ COEFF. SERVO DIFF. EQUA.
IF INPUT TO
RATE GYRO
$-K_g[D][h_F]$
$-K_g[D][h_R]$
ACCELEROMETER
$-K_a[H][h_F]$
$-K_a[H][h_R]$
DETERMINE MAXIMUM ORDER OF SERVO ELEMENTS
FORM X,Y,Z MATRICES
CALL FORMAB
FORMULATE [A] & [B]
STORE ON FILE 2
IF NA
= 0
> 0
CALL AIC
READ AIC's
SEPARATE REAL AND IMAG.
CALL GENMAT
GENERALIZED AERO FORCES
STORE ON FILE 1
See next page.


From preceding page
IF NA
> 0
= 0
CALL HPAS
CALL HPSE
CALL INTERP
INTERPOLATION MATRICES
READ MACH NO. DETERMINE VELOCITY
MACH. NO. INTERPOLATION IF REQUIRED
CALL SPLINE
GAIN OPTION
STORE ALTERED [A], [B] ON FILE 3
INTERPOLATION OF AERO FORCES FOR EACH MODE
CALL SPLINE
CALL SOLV
EIGENVALUE SOLUTION
LINING UP THE FREQUENCIES
ESTIMATE NEW OMEGA
DETERMINE REDUCED FREQ. FOR NEXT MODE
CALL RESULT
STORE STAB. RESULTS FOR EACH MODE
IF EIGENVECTORS ARE REQUESTED
No
Yes
CALL EGNVCT
PRINT STABILITY RESULTS, EIGENVECTORS DEFLECTIONS
END
READ [A] & [B] FROM FILE 2
IF GAIN VARIES
NO
YES
DIVIDE OUTPUT COEFFICIENTS BY GAIN FACTOR
CALL SOLV
GENERATE DYNAMIC MATRIX-EIGENVALUE PROBLEM
CALL IMSL
CALL EIGRF
EIGENVALUE SOLUTION
CALL RESULT
CALCULATE STABILITY RESULTS
CALL EGNVCT
CALCULATE EIGENVECTORS
PRINT STABILITY RESULTS, EIGENVECTORS, DEFLECTIONS

SECTION VI

PROGRAM LISTING

```
      PROGRAM MPASES(INPUT,OUTPUT,TAPE1,TAPE2,TAPE3,TAPE4,TAPE5=INPUT,
     1 TAPE6=OUTPUT)
C     MPASES - MODIFIED PASES, AERO-SERVO-ELASTIC STABILITY ANALYSIS
C     DYNAMIC STORAGE ALLOCATION USED
C     IMSL EIGENVALUE PACKAGE USED
C
      COMMON/CONTRL/ MCODE,NU,N1,N2,IC,NA,ND,NG,NGS,K1,K2,NPM,NMM,NVEC,
     1 NAM
      COMMON/INPUT/MU(5),AM(5),XR(10),DENS(5),SOS(5),GAIN(25)
      COMMON/AERO/SS,CBAR,S,SR,UM
      COMMON Z(10000)
      DIMENSION TITLE(30)
      EQUIVALENCE (MU(1),TITLE(1))
C
      CALL IMSL
C     READ INPUT AND PRINT
    1 READ(5,501) TITLE
      IF(EOF(5))2,4
    2 STOP
    4 CONTINUE
      REWIND 1
      REWIND 2
      REWIND 3
      WRITE(6,601) TITLE
      READ(5,502) MCODE,NOP,NPM,NMM,NC,NSE,NA,NM,NK,ND,MODE,NG,NGS,NAM
      READ(5,501) GAMMA
      NF=NPM+NMM
      NPODE=NPM+NC
      NT=NMODE+NSE
      NSERV=(NSE+NC)*5
      IF(NA.EQ.0) GO TO 5
      READ(5,503) SS,CBAR,S,UM
      BR=CBAR/2.
      READ(5,503) (AM(I),I=1,NM)
      READ(5,503) (XR(I),I=1,NK)
      READ(5,503) (DENS(I),SOS(I),I=1,ND)
      READ(5,502) (MU(I),I=1,ND)
      WRITE(6,602)
      GO TO 10
    5 WRITE(6,603)
   10 WRITE(6,604) NOF,NPM,NMM,NC,NSE,GAMMA,MODE
      IF(NA.EQ.0) GO TO 15
      WRITE(6,605) ND,NM,NR,MR,SS,S,UM
   15 IF(NGS.EQ.0) GO TO 25
      READ(5,503) (GAIN(I),I=1,NG)
      IF(NGS.LT.0) GO TO 20
      WRITE(6,606) NG,NGS
      GO TO 25
   20 INGS=IABS(NGS)
      WRITE(6,607) NG,INGS
C     DYNAMIC DIMENSIONING
   25 I1=1
      NF=NPM
      IF(NPM.EQ.0) NF=1
      MM=NMM
      IF(NMM.EQ.0) MM=1
      NOF=NOF
```

```
      IF(NPM.EQ.0) NOP=1                                                MAIN0560
      M3=NOP                                                            MAIN0570
      IF(NMM.EQ.0) NOP=1                                                MAIN0580
      M1=MAX0(NPM,NMM)                                                  MAIN0590
      M2=MAX0(M1,NC)                                                    MAIN0600
      I2=I1+NOP*NFM                                                     MAIN0610
      I3=I2+NOP*NMM                                                     MAIN0620
      I4=I3+NOP*NC                                                      MAIN0630
      I5=I4+NP*NMODE                                                    MAIN0640
      I6=I5+NPM                                                         MAIN0650
      I7=I6+NFM                                                         MAIN0660
      I8=I7+NPM                                                         MAIN0670
      I9=I8+NPM                                                         MAIN0680
      I10=I9+NOP*NOP                                                    MAIN0690
      I11=I10+NI*NOP                                                    MAIN0700
      J1=I11+M2*M2                                                      MAIN0710
C     FORM GENERALIZED MASS MATRICES                                    MAIN0720
      CALL MASS(Z(I1),Z(I2),Z(I3),Z(I4),Z(I5),Z(I6),Z(I7),Z(I8),Z(I9),  MAIN0730
     1 Z(I10),Z(I11),NPM,NMM,NC,NOP,NOM,NOP,NPM,NMODE,M1,M2)            MAIN0740
      I8=I7+NI*NI                                                       MAIN0750
      I9=I8+NI*NI                                                       MAIN0760
      I10=I9+NI*NI                                                      MAIN0770
      I11=I10+NSE*NI                                                    MAIN0780
      I12=I11+NSE*NI                                                    MAIN0790
      I13=I12+NSE*NI                                                    MAIN0800
      I14=I13+NSE*2                                                     MAIN0810
      I15=I14+NSE*NSERV                                                 MAIN0820
      I16=I15+NSERV*2                                                   MAIN0830
      I17=I16+NSERV                                                     MAIN0840
      I18=I17+NSERV                                                     MAIN0850
      I19=I18+NSERV                                                     MAIN0860
      I20=I19+NOP                                                       MAIN0870
      I21=I20+NOM                                                       MAIN0880
      J2=I21+NOM                                                        MAIN0890
C     READ COEFFICIENTS FROM SERVO DIFFERENTIAL EQUATIONS               MAIN0900
      N0=0                                                              MAIN0910
      N1=0                                                              MAIN0920
      N2=0                                                              MAIN0930
      CALL SERVO(Z(I1),Z(I2),Z(I7),Z(I8),Z(I9),Z(I10),Z(I11),Z(I12),    MAIN0940
     1 Z(I13),Z(I14),Z(I15),Z(I16),Z(I17),Z(I18),Z(I19),Z(I20),Z(I21),  MAIN0950
     2 NPM,NMM,NSE,NOP,NSERV,NI,NOF,NOM,NC)                             MAIN0960
      NT=NI+NC                                                          MAIN0970
      IF(NODE.GT.NT) NODE=NT                                            MAIN0975
C     FORM A AND B MATRICES                                             MAIN0980
      I10=I9+NT*NT                                                      MAIN0990
      J3=I10+NT*NT                                                      MAIN1000
      CALL FORMAB(Z(I4),Z(I5),Z(I6),Z(I7),Z(I8),Z(I9),Z(I10),Z(I11),    MAIN1010
     1 Z(I12),Z(I13),Z(I14),NPM,NMODE,NPM,NI,NSE,NT)                    MAIN1020
      IF(NA.EQ.0) GO TO 30                                              MAIN1030
      I5=I4+2*NOP*NOP                                                   MAIN1040
      I6=I5+NOP*NOP                                                     MAIN1050
      I7=I6+NOP*NOP                                                     MAIN1060
      I8=I7+NPM*NMODE                                                   MAIN1070
      I9=I8+NPM*NMODE                                                   MAIN1080
      I10=I9+NI*NOF                                                     MAIN1090
      J4=I10+M2*M2                                                      MAIN1100
C     READ AIC MATRICES                                                 MAIN1110
      CALL AIC(Z(I1),Z(I2),Z(I3),Z(I4),Z(I5),Z(I6),Z(I7),Z(I8),Z(I9),   MAIN1120
     1 Z(I10),NOP,NPM,NMODE,M1,NPM,NOP,NMM,NUM,NC,M2,NM,NK)             MAIN1130
   30 I2=I1+NT*NT                                                       MAIN1140
      I3=I2+NT*NT                                                       MAIN1150
      I4=I3+NODE*7                                                      MAIN1160
      I5=I4+2*NODE*NT                                                   MAIN1170
      I6=I5+NT*NT                                                       MAIN1180
      I7=I6+NT*NT                                                       MAIN1190
```

```
      I8=I7+2*NTT                                                       MAIN1200
      I9=I8+NTT                                                         MAIN1210
      I10=I9+NTT                                                        MAIN1220
      I11=I10+NTT                                                       MAIN1230
      I12=I11+NTT                                                       MAIN1240
      I13=I12+NTT*3                                                     MAIN1250
      IDEF=I13+NDF*2                                                    MAIN1260
      IMSI=IDEF+NDF*NMODE                                               MAIN1265
      IV=IMSI+NTT                                                       MAIN1266
      IF(NA.NE.0) GO TO 35                                              MAIN1270
      J4=IV+2                                                           MAIN1280
      CALL MPSE(Z(I1),Z(I2),Z(I3),Z(I4),Z(I5),Z(I6),Z(I7),Z(I8),Z(I9),  MAIN1290
     1 Z(I10),Z(I11),Z(I12), GAIN ,NTT,MODE,NMODE,NFR,GAMMA,NDF,Z(I13), MAIN1300
     2 Z(IDEF),Z(IMSI),Z(IV))                                           MAIN1310
      JTOT=MAX0(J1,J2,J3,J4)                                            MAIN1320
      WRITE(6,610) JTOT                                                 MAIN1330
      GO TO 1                                                           MAIN1340
   35 I14=I13+(NM+1)*NM                                                 MAIN1350
      I15=I14+(NK+1)*NK                                                 MAIN1360
      M3=MAX0(NM,NK)                                                    MAIN1370
      I16=I15+(M3+1)*M3                                                 MAIN1380
      I17=I16+(M3+1)*(M3+1)                                             MAIN1390
      I18=I17+10                                                        MAIN1400
      I19=I18+NFR*NMODE*NK                                              MAIN1410
      I20=I19+NFR*NMODE*NK                                              MAIN1420
      I21=I20+NFR*NMODE*NM                                              MAIN1430
      I22=I21+NFR*NMODE*NM                                              MAIN1440
      I23=I22+NFR*NMODE                                                 MAIN1450
      I24=I23+NFR*NMODE                                                 MAIN1460
      I25=I24+NM                                                        MAIN1470
      I26=I25+NK                                                        MAIN1480
      I27=I26+M3+1                                                      MAIN1490
      IDEF=I27+NDF*2                                                    MAIN1500
      IMSI=IDEF+NDF*NMODE                                               MAIN1505
      IV=IMSI+NTT                                                       MAIN1506
      J5=IV+2                                                           MAIN1510
      CALL MPAS(Z(I1),Z(I2),Z(I3),Z(I4),Z(I5),Z(I6),Z(I7),Z(I8),Z(I9),  MAIN1520
     1 Z(I10),Z(I11),Z(I12),Z(I13),Z(I14),Z(I15),Z(I16),Z(I17),Z(I18),  MAIN1530
     2 Z(I19),Z(I20),Z(I21),Z(I22),Z(I23),Z(I24),Z(I25),Z(I26),NTT,NFR, MAIN1540
     3 NMODE,NM,NM+1,NK,NK+1,MODE,M3,M3+1,GAMMA,NDF,Z(I27),Z(IDEF),     MAIN1550
     4 Z(IMSI),Z(IV))                                                   MAIN1555
      JTOT=MAX0(J1,J2,J3,J4,J5)                                         MAIN1560
      WRITE(6,610) JTOT                                                 MAIN1570
      GO TO 1                                                           MAIN1580
  501 FORMAT(18A4)                                                      MAIN1590
  502 FORMAT(16I5)                                                      MAIN1600
  503 FORMAT(6E12.0)                                                    MAIN1610
  601 FORMAT(1H1,25X,18A4//26X,18A4//)                                  MAIN1620
  602 FORMAT(1H0 40X,37HAERO-SERVO-ELASTIC STABILITY ANALYSIS //)       MAIN1630
  603 FORMAT(1H0 43X,32HSERVO ELASTIC STABILITY ANALYSIS // )           MAIN1640
  604 FORMAT(1H0 44X, I2 ,19H DEGREES OF FREEDOM/49X,I2,15H FLEXIBLE MODMAIN1650
     1ES/49X,I2,17H RIGID BODY MODES/49X,I2,17H CONTROL SURFACES/ 48X,  MAIN1660
     2I3,15H SERVO ELEMENTS//42X,25HSHIFT EIGENVALUE(GAMMA) = 1PE11.3   MAIN1670
     3 //48X,I3,15H MODES REQUESTED //)                                 MAIN1680
  605 FORMAT(1H0,49X,I1,20H ALTITUDE VARIATIONS//35X,17HAIC MATRICES FORMAIN1690
     1 ,I1,18H MACH NUMBERS AND ,I2,20H REDUCED FREQUENCIES//42X,       MAIN1700
     2 22HREFERENCE SEMI-CHORD = 1PE12.5,3H FT/53X,11HSEMI-SPAN =       MAIN1710
     3 1PE12.5,3H FT/50X,14HSURFACE AREA = 1PE12.5,6H SQ FT//41X,       MAIN1720
     4 23HMACH NUMBER DEVIATION = E12.5 //)                             MAIN1721
  606 FORMAT(1H0,37X,I2,41H GAIN FACTOR VARIATIONS FOR SERVO ELEMENT I3 MAIN1730
     1//)                                                               MAIN1740
  607 FORMAT(1H0,37X,I2,43H GAIN FACTOR VARIATIONS FOR CONTROL SURFACE  MAIN1750
     1I2 //)                                                            MAIN1760
  610 FORMAT(1H1,2X,38HMINIMUM BLANK COMMON LENGTH REQUIRED = I6,1H.,/3X,MAIN1770
     1 43HBASED ON INPUT DATA AND ANALYSES REQUESTED.)                  MAIN1780
      END                                                               MAIN1790
```

```
      SUBROUTINE MASS(FMODE,RMODE,CMODE,GENM,SK,CF,GF,FREQ,WT,TM,FRC,
     1 NFM,NRM,NC,NRF,NOR,NOF,NFR,NMODE,M1,M2)
C     FORM GENERALIZED MASS MATRIX
      COMMON/CONTROL/MCODE,N0,N1,N2,NTC,NA,NO,NG,NGS,K1,K2,NFM,NRM,NVEC,
     1 NAB
      DIMENSION FMODE(NOF,NFM),RMODE(NOR,NRM),CMODE(NOF,NC),
     1 GENM(NFR,NMODE),SK(NFM),CF(NFM),GF(NFM),FREQ(NFM),WT(NOF,NOF),
     2 TM(M1,NOF),FRC(M2,M2)
C
C     READ WEIGHT MATRIX
      REWIND 8
      IF(MCODE.EQ.2) GO TO 35
      DO 30 I=1,NOF
      DO 30 J=1,NOF
   30 WT(I,J)=0.0
      READ(5,503) (WT(I,I),I=1,NOF)
      GO TO 45
   35 DO 40 I=1,NOF
      READ(5,503) (WT(I,J),J=I,NOF)
      IF(I.EQ.NOF) GO TO 40
      II=I+1
      DO 36 J=II,NOF
   36 WT(J,I)=WT(I,J)
   40 CONTINUE
   45 WRITE(6,608)
      DO 50 I=1,NOF
   50 WRITE(6,609) I,(WT(I,J),J=I,NOF)
      DO 55 I=1,NOF
      DO 55 J=1,NOF
   55 WT(I,J)=WT(I,J)/32.174
C     READ MODES AND PRINT
      WRITE(6,610)
      DO 60 I=1,NC
      READ(5,503) (CMODE(J,I),J=1,NOF)
   60 WRITE(6,611) I,(CMODE(J,I),J=1,NOF)
      IF(NFM.EQ.0) GO TO 70
      READ(5,503) (GF(I),I=1,NFM)
      READ(5,503) (FNEQ(I),I=1,NFM)
      WRITE(6,612)
      DO 65 I=1,NFM
      READ(5,503) (FMODE(J,I),J=1,NOF)
      WRITE(6,613) I,FNEQ(I),GF(I),(FMODE(J,I),J=1,NOF)
   65 FREQ(I)=FNEQ(I)*6.2831853
      WRITE(8)((FMODE(I,J),I=1,NOF),J=1,NFM)
   70 IF(NRM.EQ.0) GO TO 80
      WRITE(6,614)
      DO 75 I=1,NRM
      READ(5,503) (RMODE(J,I),J=1,NOF)
   75 WRITE(6,611) I,(RMODE(J,I),J=1,NOF)
      WRITE(8)((RMODE(I,J),I=1,NOF),J=1,NRM)
   80 CALL GENMAT(WT,GENM,TM,FRC,FMODE,RMODE,CMODE,NOF,NOR,NOF,NC,NFR,
     1 M1,M2)
      WRITE(8)((CMODE(I,J),I=1,NOF),J=1,NC)
C     FORM GENERALIZED STIFFNESS AND DAMPING MATRICES
      IF(NFM.EQ.0) RETURN
      DO 85 I=1,NFM
      SK(I)=FREQ(I)*FREQ(I)*GENM(I,I)
   85 CF(I)=GF(I)*FREQ(I)*GENM(I,I)
      RETURN
  503 FORMAT(6E12.8)
  608 FORMAT(1H0 2X,31HUPPER TRIANGLE OF WEIGHT MATRIX //)
  609 FORMAT(1H0 2X,3HROW I3 /(3X,1P8E15.5))
  610 FORMAT(/// 3X,32HRIGID BODY CONTROL SURFACE MODES //)
  611 FORMAT(1H0 2X,4HMODE,I2 //(3X,1P8E15.5))
  612 FORMAT(/// 3X,14HFLEXIBLE MODES //)
  613 FORMAT(1H0 2X,4HMODE,I2,14H - FREQUENCY = F12.3,4H CPS/12X,
     132HSTRUCTURAL DAMPING COEFFICIENT = F6.3 //(3X,1P8E15.5))
  614 FORMAT(/// 3X,16HRIGID BODY MODES //)
      END
```

```
      SUBROUTINE SERVO(FMODE,RMODE,X0,X1,X2,Y0,Y1,Z0,KO,NOR,NL,C0,C1,    SERV0010
     1 C2,D,DFM,DRM,NFM,NRM,NSE,NOF,NSERV,NT,NOF,NOR,NC)                 SERV0020
C     SERVO - SERVO DIFFERENTIAL EQUATIONS                                SERV0030
      COMMON/CONTROL/MCODE,N0,N1,N2,NTC,NA,ND,NG,NGS,K1,K2,NFM,NRM,NVEC,SERV0040
     1 NAB                                                                SERV0050
      DIMENSION FMODE(NOF,NFM),RMODE(NOR,NRM),X0(NT,NT),X1(NT,NT),        SERV0060
     1 X2(NT,NT),Y0(NT ,NSE),Y1(NT ,NSE),Z0(NT ,NSE),D(1,NOF),            SERV0070
     2 DFM(1,NFM),DRM(1,NRM),KO(NSE,2),NOR(NSE,NSERV),NL(NSERV,2),        SERV0080
     3 C0(NSERV),C1(NSERV),C2(NSERV)                                      SERV0090
C                                                                         SERV0100
      NFR=NFM+NRM                                                         SERV0110
      NMODE=NFR+NC                                                        SERV0120
      DO 7 I=1,NT                                                         SERV0130
      DO 5 J=1,NT                                                         SERV0140
      X0(I,J)=0.0                                                         SERV0150
      X1(I,J)=0.0                                                         SERV0160
    5 X2(I,J)=0.0                                                         SERV0170
      DO 6 JJ=1,NSE                                                       SERV0180
      Y0(I,JJ)=0.0                                                        SERV0190
      Y1(I,JJ)=0.0                                                        SERV0200
    6 Z0(I,JJ)=0.0                                                        SERV0210
    7 CONTINUE                                                            SERV0220
C                                                                         SERV0230
C     READ COEFFICIENTS FROM SERVO DIFFERENTIAL EQUATIONS                 SERV0240
C     PRINT CONTROL SYSTEM DESCRIPTION                                    SERV0250
      WRITE(6,600)                                                        SERV0260
      L=0                                                                 SERV0270
      IND=0                                                               SERV0275
      DO 25 I=1,NC                                                        SERV0280
      IC=NFR+I                                                            SERV0290
      READ(5,503) X0(IC,IC),X1(IC,IC),X2(IC,IC)                           SERV0300
      WRITE(6,601) I,I,X2(IC,IC),X1(IC,IC),X0(IC,IC)                      SERV0310
      READ(5,502) INC                                                     SERV0320
      DO 20 J=1,INC                                                       SERV0330
      L=L+1                                                               SERV0340
      READ(5,504) K,NO,C0(L),C1(L),C2(L)                                  SERV0350
      WRITE(6,602) K,C2(L),C1(L),C0(L)                                    SERV0360
      DO 15 KK=1,NSE                                                      SERV0370
      IF(KK.EQ.K) GO TO 10                                                SERV0380
      NOR(KK,L)=0                                                         SERV0390
      GO TO 15                                                            SERV0400
   10 NOR(KK,L)=NO                                                        SERV0410
   15 CONTINUE                                                            SERV0420
      NL(L,1)=I                                                           SERV0430
      NL(L,2)=K                                                           SERV0440
   20 CONTINUE                                                            SERV0450
   25 CONTINUE                                                            SERV0460
      NINC=L                                                              SERV0470
      DO 115 J=1,NSE                                                      SERV0480
      L=L+1                                                               SERV0490
      READ(5,504) I,NO,C0(L),C1(L),C2(L)                                  SERV0500
      WRITE(6,603) I,I,C2(L),C1(L),C0(L)                                  SERV0510
      NL(L,1)=I                                                           SERV0520
      NL(L,2)=I                                                           SERV0530
      DO 35 KK=1,NSE                                                      SERV0540
      IF(KK.EQ.I) GO TO 30                                                SERV0550
      NOR(KK,L)=0                                                         SERV0560
      GO TO 35                                                            SERV0570
   30 NOR(KK,L)=NO                                                        SERV0580
   35 CONTINUE                                                            SERV0590
      READ(5,502) INS                                                     SERV0600
      IF(INS.EQ.0) GO TO 115                                              SERV0610
      DO 110 M=1,INS                                                      SERV0620
      L=L+1                                                               SERV0630
      READ(5,504) K,NO,C0(L),C1(L),C2(L)                                  SERV0640
      IF(K.LT.0) GO TO 100                                                SERV0650
      IF(NO.LT.0) GO TO 50                                                SERV0660
      WRITE(6,602) K,C2(L),C1(L),C0(L)                                    SERV0670
      DO 45 KK=1,NSE                                                      SERV0680
      IF(KK.EQ.K) GO TO 40                                                SERV0690
      NOR(KK,L)=0                                                         SERV0700
      GO TO 45                                                            SERV0710
   40 NOR(KK,L)=NO                                                        SERV0720
   45 CONTINUE                                                            SERV0730
      NL(L,1)=I                                                           SERV0740
      NL(L,2)=K                                                           SERV0750
      GO TO 110                                                           SERV0760
C     INPUT TO RATE GYRO OR ACCELEROMETER                                 SERV0770
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```
   50 NL(L,1)=0                                                        SERV0780
      NL(L,2)=0                                                        SERV0790
      IF(NO.EQ.(-1)) GO TO 55                                          SERV0800
      WRITE(6,604) C0(L)                                               SERV0810
      GO TO 60                                                         SERV0820
   55 WRITE(6,605) C0(L)                                               SERV0830
   60 DO 65 KK=1,NSF                                                   SERV0840
   65 NOR(KK,L)=0                                                      SERV0850
      INO=1                                                            SERV0855
      READ(5,503) (D(1,JJ),JJ=1,NOF)                                   SERV0860
      WRITE(1) I,(D(1,JJ),JJ=1,NOF)                                    SERV0865
      DO 70 JJ=1,NOF                                                   SERV0870
   70 D(1,JJ)=C0(L)*D(1,JJ)                                            SERV0880
      IR=NMODF+I                                                       SERV0890
      IF(NFM.EQ.0) GO TO 85                                            SERV0900
      CALL MATMPL(D,FMODE,DFM,1,NOF,1,1,NFM,NOF,1)                     SERV0910
      DO 80 JJ=1,NFM                                                   SERV0920
      IF(NO.EQ.(-1)) GO TO 75                                          SERV0930
      X2(IR,JJ)=X2(IR,JJ)+DFM(1,JJ)                                    SERV0940
      GO TO 80                                                         SERV0950
   75 X1(IR,JJ)=X1(IR,JJ)+DFM(1,JJ)                                    SERV0960
   80 CONTINUE                                                         SERV0970
   85 IF(NRM.EQ.0) GO TO 110                                           SERV0980
      CALL MATMPL(D,RMODE,DRM,1,NOR,1,1,NRM,NOF,1)                     SERV0990
      DO 95 JJ=1,NRM                                                   SERV1000
      IC=NFM+JJ                                                        SERV1010
      IF(NO.EQ.(-1)) GO TO 90                                          SERV1020
      X2(IR,IC)=X2(IR,IC)+DRM(1,JJ)                                    SERV1030
      GO TO 95                                                         SERV1040
   90 X1(IR,IC)=X1(IR,IC)+DRM(1,JJ)                                    SERV1050
   95 CONTINUE                                                         SERV1060
      GO TO 110                                                        SERV1070
C     INPUT FROM CONTROL SURFACE                                       SERV1080
  100 NL(L,1)=0                                                        SERV1090
      NL(L,2)=0                                                        SERV1100
      IAK=IABS(K)                                                      SERV1110
      WRITE(6,606) IAK,C2(L),C1(L),C0(L)                               SERV1120
      DO 105 KK=1,NSF                                                  SERV1130
  105 NOR(KK,L)=0                                                      SERV1140
      IR=NMODF+I                                                       SERV1150
      IC=NFM+IABS(K)                                                   SERV1160
      X0(IR,IC)=C0(L)                                                  SERV1170
      X1(IR,IC)=C1(L)                                                  SERV1180
      X2(IR,IC)=C2(L)                                                  SERV1190
  110 CONTINUE                                                         SERV1200
  115 CONTINUE                                                         SERV1210
      IF(INO.EQ.0) GO TO 118                                           SERV1211
      ENDFILE 1                                                        SERV1212
      REWIND 1                                                         SERV1213
      WRITE(6,610)                                                     SERV1214
  116 READ(1) I,(D(1,JJ),JJ=1,NOF)                                     SERV1215
      IF(EOF(1)) 117,111                                               SERV1216
  111 CONTINUE                                                         SERV1217
      WRITE(6,611) I,(D(1,JJ),JJ=1,NOF)                                SERV1218
      GO TO 116                                                        SERV1219
  117 REWIND 1                                                         SERV1220
  118 CONTINUE                                                         SERV1221
C     DETERMINE MAXIMUM ORDER OF SERVO ELEMENTS                        SERV1222
      DO 140 K=1,NSF                                                   SERV1230
      DO 120 LL=1,L                                                    SERV1240
      IF(NOR(K,LL).EQ.2) GO TO 135                                     SERV1250
  120 CONTINUE                                                         SERV1260
      DO 125 LL=1,L                                                    SERV1270
      IF(NOR(K,LL).EQ.1) GO TO 130                                     SERV1280
  125 CONTINUE                                                         SERV1290
      KO(K,1)=0                                                        SERV1300
      N0=N0+1                                                          SERV1310
      KO(K,2)=N0                                                       SERV1320
      GO TO 140                                                        SERV1330
  130 KO(K,1)=1                                                        SERV1340
      N1=N1+1                                                          SERV1350
      KO(K,2)=N1                                                       SERV1360
      GO TO 140                                                        SERV1370
  135 KO(K,1)=2                                                        SERV1380
      N2=N2+1                                                          SERV1390
      KO(K,2)=N2                                                       SERV1400
  140 CONTINUE                                                         SERV1410
C     FORM X, Y, Z MATRICES                                            SERV1420
      DO 170 I=1,L                                                     SERV1430
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```
      IF(NL(I,1).EQ.0) GO TO 170                                        SERV1440
      IF(I.GT.NINC) GO TO 145                                           SERV1450
      II=NFR+NL(I,1)                                                    SERV1460
      GO TO 150                                                         SERV1470
  145 II=NMODE+NL(I,1)                                                  SERV1480
  150 K=NL(I,2)                                                         SERV1490
      M=KO(K,1)                                                         SERV1500
      IF(M-1)165,160,155                                                SERV1510
  155 JJ=NMODE+KO(K,2)                                                  SERV1520
      X2(II,JJ)=C2(I)                                                   SERV1530
      X1(II,JJ)=C1(I)                                                   SERV1540
      X0(II,JJ)=C0(I)                                                   SERV1550
      GO TO 170                                                         SERV1560
  160 JJ=KO(K,2)                                                        SERV1570
      Y1(II,JJ)=C1(I)                                                   SERV1580
      Y0(II,JJ)=C0(I)                                                   SERV1590
      GO TO 170                                                         SERV1600
  165 JJ=KO(K,2)                                                        SERV1610
      Z0(II,JJ)=C0(I)                                                   SERV1620
  170 CONTINUE                                                          SERV1630
      NTC=NMODE+N2                                                      SERV1640
C     PRINT SERVO ELEMENT INFORMATION                                   SERV1650
      WRITE(6,607)                                                      SERV1660
      DO 200 I=1,NSE                                                    SERV1670
      II=NMODE+I                                                        SERV1680
      IF(KO(I,1)-1) 175,180,185                                         SERV1690
  175 KOP=2*NTC+N1+KO(I,2)                                              SERV1700
      GO TO 190                                                         SERV1710
  180 KOP=2*NTC+KO(I,2)                                                 SERV1720
      GO TO 190                                                         SERV1730
  185 KO1=NMODE+KO(I,2)                                                 SERV1740
      KO2=NTC+KO1                                                       SERV1750
      GO TO 195                                                         SERV1760
  190 WRITE(6,608) I,KO(I,1),II,KOP,KOP                                 SERV1770
      GO TO 200                                                         SERV1780
  195 WRITE(6,609) I,KO(I,1),II,KO1,KO2,KO1,KO2                         SERV1790
  200 CONTINUE                                                          SERV1800
C     SAVE FOR GAIN OPTION                                              SERV1810
      IF(NGS.EQ.0) RETURN                                               SERV1820
      K1=KO(NGS,1)                                                      SERV1830
      K2=KO(NGS,2)                                                      SERV1840
      RETURN                                                            SERV1850
  502 FORMAT(16I5)                                                      SERV1860
  503 FORMAT(6E12.0)                                                    SERV1870
  504 FORMAT(2I5,2X,3E12.0)                                             SERV1880
  600 FORMAT(1H1 50X,28HCONTROL SYSTEM DESCRIPTION///3X,                SERV1890
     125HDIFFERENTIAL EQUATION FOR,22X,8HVARIABLE,30X,12HCOEFFICIENTS,  SERV1900
     220X,11HGYRO/ACCEL./                                               SERV1910
     376X,9H2ND ORDER,4X,9H1ST ORDER,6X,7H0 ORDER,9X,11HGAIN FACTOR /)  SERV1920
  601 FORMAT(1H0 2X,13HCONTROL SURF. I3,27X,13HCONTROL SURF. I3,10X,    SERV1930
     11PE14.4,1P2E13.4)                                                 SERV1940
  602 FORMAT(1H  65X,13HSERVO ELEMENT I3,10X,1PE14.4,1P2E13.4)          SERV1950
  603 FORMAT(1H0 2X,13HSERVO ELEMENT I3,27X,13HSERVO ELEMENT I3,10X,    SERV1960
     11PE14.4,1P2E13.4)                                                 SERV1970
  604 FORMAT(1H  2X,15H(ACCELEROMETER),28X,4HBODY/46X,13HACCELERATION , SERV1980
     154X,1PE14.4)                                                      SERV1990
  605 FORMAT(1H  2X,11H(RATE GYRO),32X,4HBODY/46X,13HANGULAR RATE ,     SERV2000
     154X,1PE14.4)                                                      SERV2010
  606 FORMAT(1H  65X,13HCONTROL SURF. I3,10X,1PE14.4,1P2E13.4)          SERV2020
  607 FORMAT(/// 6X,5HSERVO,10X,7HMAXIMUM,5X,10HROW ASSIGNMENT,5X,      SERV2030
     1 17HCOLUMN ASSIGNMENT,8X,19HEIGENVECTOR ELEMENT/3X,7HELEMENT,     SERV2040
     2 10X,5HORDER,7X,13HIN A AND/OR B,7X,13HIN A AND/OR B/73X,         SERV2050
     3 8HVELOCITY,3X,12HDISPLACEMENT / )                                SERV2060
  608 FORMAT(1H 6X,I2,15X,I1,14X,I2,22X,I2,26X,I2)                      SERV2070
  609 FORMAT(1H 6X,I2,15X,I1,14X,I2,15X,I2,4H AND,I3,13X,I2,11X,I2)     SERV2080
  610 FORMAT(/// 6X,11HGYRO/ACCEL.,15X,46HINPUT DIFFERENTIATION/INTERPOLSERV2081
     1ATION ROW VECTOR /)                                               SERV2082
  611 FORMAT(1H0,2X,14HSERVO ELEMENT ,I2,2X,1P8E13.4/(21X,1P8E13.4))    SERV2083
      END                                                               SERV2090
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      SUBROUTINE FORMAB(GENM,SK,CF,X0,X1,X2,Y0,Y1,Z0,A,B,NFR,NMODE,NFM,
     1 NT,NSE,NTT)
C     FORM A AND B MATRICES
      COMMON/CONTROL/NCODE,N0,N1,N2,NTC,NA,ND,NG,NGS,K1,K2,NFM,NNM,NVEC,
     1 NAB
      DIMENSION GENM(NFR,NMODE),SK(NFM),CF(NFM),X0(NT,NT),X1(NT,NT),
     1 X2(NT,NT),Y0(NT ,NSE),Y1(NT ,NSE),Z0(NT ,NSE),A(NTT,NTT),
     2 B(NTT,NTT)
C
      DO 110 I=1,NTT
      DO 110 J=1,NTT
      A(I,J)=0.0
  110 B(I,J)=0.0
      DO 135 I=1,NT
      DO 115 J=1,NTC
      A(I,J)=X2(I,J)
      JJ=NTC+J
      A(I,JJ)=X1(I,J)
  115 B(I,JJ)=X0(I,J)
      IF(N1.EQ.0) GO TO 125
      DO 120 J=1,N1
      JJ=2*NTC+J
      A(I,JJ)=Y1(I,J)
  120 B(I,JJ)=Y0(I,J)
  125 IF(N0.EQ.0) GO TO 135
      DO 130 J=1,N0
      JJ=2*NTC+N1+J
  130 B(I,JJ)=Z0(I,J)
  135 CONTINUE
      DO 140 I=1,NTC
      II=NT+I
      JJ=NTC+I
      A(II,JJ)=1.0
  140 B(II,I)=-1.0
      IF(NFM.EQ.0) GO TO 148
      DO 145 I=1,NFM
      JJ=NTC+I
      A(I,JJ)=CF(I)
  145 B(I,JJ)=SK(I)
  148 DO 150 I=1,NFM
      DO 150 J=1,NMODE
  150 A(I,J)=GENM(I,J)
      WRITE(2) ((A(I,J),I=1,NTT),J=1,NTT),((B(I,J),I=1,NTT),J=1,NTT)
      RETURN
      END
```

```
      SUBROUTINE AIC(FMODE,HMODE,CMODE,CM,CMS,CMUM,GAM,GAI,TM,FMC,NUF,   AIC 0010
     1 NFM,NMODE,M1,NFM,NUP,NMM,NUM,NC,M2,NMM,NA)                        AIC 0020
C     FORM GENERALIZED AERO MATRICES                                    AIC 0030
      COMMON/CONTRL/ MODE,NU,N1,N2,NIC,NA,ND,NG,NGS,K1,K2,NFM,NMM,NVEC, AIC 0040
     1 NAM                                                              AIC 0050
      COMMON/INPUT/MU(5),AM(5),AK(10),DENS(5),SOS(5),GAIN(25)           AIC 0060
      COMMON/AERO/SS,CHAR,SMM,UM                                        AIC 0070
      COMPLEX CM                                                        AIC 0080
      DIMENSION FMODE(NUP,NFM),HMODE(NUM,NMM),CMODE(NUP,NC),CM(NUF,NUF),AIC 0090
     1 CMS(NUF,NUF),CMUM(NUF,NUF),GAM(NFM,NMODE),GAI(NFM,NMODE),        AIC 0100
     2 TM(M1,NUP),FMC(M2,M2)                                            AIC 0110
C                                                                       AIC 0120
C     READ AIC MATRICES                                                 AIC 0130
      DO 250 IM=1,NM                                                    AIC 0140
      DO 245 IK=1,NK                                                    AIC 0150
      WRITE(6,610) AM(IM),AK(IK)                                        AIC 0160
      NPAN=NNA                                                          AIC 0170
      IF(NPAN.EQ.1) GO TO 220                                           AIC 0180
      DO 200 I=1,NUP                                                    AIC 0190
      DO 200 J=1,NUP                                                    AIC 0200
  200 CM(I,J)=0.0                                                       AIC 0210
      MM=1                                                              AIC 0220
      DO 215 N=1,NPAN                                                   AIC 0230
      READ(5,502) NS,NZERO                                              AIC 0240
      IF(NZERO.EQ.1) GO TO 210                                          AIC 0250
      MT=MM+NS-1                                                        AIC 0260
      DO 205 I=MM,MT                                                    AIC 0270
  205 READ(5,503) (CM(I,J),J=MM,MT)                                     AIC 0280
  210 MM=MM+NS                                                          AIC 0290
  215 CONTINUE                                                          AIC 0300
      GO TO 230                                                         AIC 0310
  220 DO 225 I=1,NUP                                                    AIC 0320
  225 READ(5,503) (CM(I,J),J=1,NUP)                                     AIC 0330
  230 DO 235 I=1,NUP                                                    AIC 0340
  235 WRITE(6,617) (CM(I,J),J=1,NUP)                                    AIC 0350
C     SEPARATE REAL AND IMAGINARY PARTS OF AICS                         AIC 0360
      CONREF=AK(IK)**2*(CHAR*SS/S)                                      AIC 0370
      CONIM=AK(IK)*(CHAR*SS/S)                                          AIC 0380
      DO 240 I=1,NUP                                                    AIC 0390
      DO 240 J=1,NUP                                                    AIC 0400
      CMS(I,J)=CONREF*REAL(CM(I,J))                                     AIC 0410
  240 CMUM(I,J)=CONIM*AIMAG(CM(I,J))                                    AIC 0420
C     FORM GENERALIZED AERO FORCES                                      AIC 0430
      CALL GENMAT(CMS,GAM,TM,FMC,FMODE,HMODE,CMODE,NUF,NUM,NUF,NC,NFM,  AIC 0440
     1 M1,M2)                                                           AIC 0450
      CALL GENMAT(CMUM,GAI,TM,FMC,FMODE,HMODE,CMODE,NUF,NUM,NUF,NC,NFM, AIC 0460
     1 M1,M2)                                                           AIC 0470
      WRITE(1) ((GAM(I,J),I=1,NFM),J=1,NMODE),((GAI(I,J),I=1,NFM),J=1,  AIC 0480
     1 NMODE)                                                           AIC 0490
  245 CONTINUE                                                          AIC 0500
  250 CONTINUE                                                          AIC 0510
      RETURN                                                            AIC 0520
  502 FORMAT(16I5)                                                      AIC 0530
  503 FORMAT(6E12.0)                                                    AIC 0540
  610 FORMAT(1H1,1X,26HAIC MATRIX FOR MACH NUMBER = 1PE11.4,            AIC 0550
     1 24H AND REDUCED FREQUENCY = 1PE13.6 /)                           AIC 0560
  617 FORMAT(1H /(1X,(2X,1P2E15.5)))                                    AIC 0570
      END                                                               AIC 0580
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      SUBROUTINE MPSE(A,B,STAB,VEC,C,BC,E,WM,WI,IANA,BINV,INDEX,GAIN,
     1 NTT,MODE,NMODE,NFM,GAMMA,NDF,DEFL,XMODE,WR,V)
C     SERVO ELASTIC ANALYSIS
      COMMON/CONTRL/ MCODE,NO,N1,N2,NTC,NA,ND,NG,NGS,K1,K2,NFM,NRM,NVEC,
     1 NAR
      DIMENSION A(NTT,NTT),B(NTT,NTT),STAB(MODE,7),VEC(MODE,NTT),
     1 C(NTT,NTT),BC(NTT,NTT),E(NTT),WM(NTT),WI(NTT),IANA(NTT),
     2 BINV(NTT,1),INDEX(NTT,3),GAIN(1),DEFL(NDF),XMODE(NDF,NMODE)
     3 ,WR(NTT),V(1,1)
      COMPLEX VEC,E,DEFL,ANORM,V
C
C     GAIN FACTOR OPTION
      NVEC=1
      DO 185 IG=1,NG
      REWIND 2
      READ(2) ((A(I,J),I=1,NTT),J=1,NTT),((B(I,J),I=1,NTT),J=1,NTT)
      IF(NGS) 120,125,101
  101 L=K1
      II=NMODE+NGS
      IF(L-1) 115,110,105
  105 JJ=NMODE+K2
      J=NTC+JJ
      IF(GAIN(IG).EQ.0.) GO TO 108
      A(II,JJ)=A(II,JJ)/GAIN(IG)
      A(II,J)=A(II,J)/GAIN(IG)
      B(II,J)=B(II,J)/GAIN(IG)
      GO TO 125
  108 A1=A(II,JJ)
      A2=A(II,J)
      B1=B(II,J)
      DO 109 LL=1,NTT
      A(II,LL)=0.
  109 B(II,LL)=0.
      A(II,JJ)=A1
      A(II,J)=A2
      B(II,J)=B1
      GO TO 125
  110 JJ=K2
      J=2*NTC+JJ
      IF(GAIN(IG).EQ.0.) GO TO 113
      A(II,J)=A(II,J)/GAIN(IG)
      B(II,J)=B(II,J)/GAIN(IG)
      GO TO 125
  113 A1=A(II,J)
      B1=B(II,J)
      DO 114 LL=1,NTT
      A(II,LL)=0.
  114 B(II,LL)=0.
      A(II,J)=A1
      B(II,J)=B1
      GO TO 125
  115 JJ=K2
      J=2*NTC+N1+JJ
      IF(GAIN(IG).EQ.0.) GO TO 118
      B(II,J)=B(II,J)/GAIN(IG)
      GO TO 125
  118 B1=B(II,J)
      DO 119 LL=1,NTT
  119 B(II,LL)=0.
      B(II,J)=B1
      GO TO 125
  120 II=NFM+IABS(NGS)
      JJ=NTC+NFM+IABS(NGS)
      IF(GAIN(IG).EQ.0.) GO TO 121
      A(II,II)=A(II,II)/GAIN(IG)
      A(II,JJ)=A(II,JJ)/GAIN(IG)
      B(II,JJ)=B(II,JJ)/GAIN(IG)
      GO TO 125
  121 A1=A(II,II)
      A2=A(II,JJ)
      B1=B(II,JJ)
      DO 122 LL=1,NTT
      A(II,LL)=0.
  122 B(II,LL)=0.
      A(II,II)=A1
      A(II,JJ)=A2
      B(II,JJ)=B1
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  129 EK=0.                                                            MPSE0405
      EB=0.                                                            MPSE0410
      IE=0.                                                            MPSE0420
C     EIGENVALUE SOLUTION                                              MPSE0430
      CALL SOLVIT(EK,EB,IE,GAMMA,A,B,C,BC,HH,HI,E,IANA,HINV,INDEX,STAB,   MPSE0440
     1 VEC,NIT,NDOF,IMODE,NR,V)                                        MPSE0450
C     PRINT STABILITY RESULTS                                          MPSE0460
      WRITE(6,618)                                                     MPSE0470
      IF(NGS) 160,165,155                                              MPSE0480
  155 WRITE(6,619) GAIN(IG),NGS                                        MPSE0490
      GO TO 165                                                        MPSE0500
  160 INGS=IABS(NGS)                                                   MPSE0510
      WRITE(6,620) GAIN(IG),INGS                                       MPSE0520
  165 WRITE(6,622)                                                     MPSE0530
      DO 175 I=1,IMODE                                                 MPSE0540
      IF(STAB(I,2).EQ.0.) GO TO 170                                    MPSE0550
      WRITE(6,623) I,(STAB(I,J),J=1,4),(STAB(I,J),J=6,7)               MPSE0560
      GO TO 175                                                        MPSE0570
  170 WRITE(6,625) I,(STAB(I,J),J=1,6),STAB(I,7)                       MPSE0580
  175 CONTINUE                                                         MPSE0590
C     PRINT EIGENVECTORS                                               MPSE0600
      WRITE(6,615)                                                     MPSE0610
      DO 176 I=1,IMODE                                                 MPSE0620
  176 WRITE(6,624)I,(VEC(I,J),J=1,NIT)                                 MPSE0630
C     PRINT STRUCTURAL DEFLECTIONS                                     MPSE0640
      NF=INU 6                                                         MPSE0650
      IF(NFM.EQ.0) GO TO 177                                           MPSE0660
      READ(6)((XMODE(I,J),I=1,NOF),J=1,NFM)                            MPSE0670
  177 NF1=NFM+1                                                        MPSE0680
      IF(NPM.EQ.0) GO TO 178                                           MPSE0690
      READ(6)((XMODE(I,J),I=1,NOF),J=NF1,NFM)                          MPSE0700
  178 NFM1=NFM+1                                                       MPSE0710
      READ(6)((XMODE(I,J),I=1,NOF),J=NFM1,NMODE)                       MPSE0720
      WRITE(6,626)                                                     MPSE0730
      DO 184 IMO=1,IMODE                                               MPSE0740
      DO 181 I=1,NOF                                                   MPSE0750
      DEFL(I)=0.0                                                      MPSE0760
      DO 180 J=1,NMODE                                                 MPSE0770
      K=NTC+J                                                          MPSE0780
  180 DEFL(I)=DEFL(I)+XMODE(I,J)*VEC(IMO,K)                            MPSE0790
  181 CONTINUE                                                         MPSE0800
      TST=0.0                                                          MPSE0810
      DO 182 I=1,NOF                                                   MPSE0820
      ANORM=CABS(DEFL(I))                                              MPSE0830
      IF(TST.GT.ANORM) GO TO 182                                       MPSE0840
      L=I                                                              MPSE0850
      TST=ANORM                                                        MPSE0860
  182 CONTINUE                                                         MPSE0870
      XNORM=DEFL(L)                                                    MPSE0880
      ANORM=CABS(XNORM)                                                MPSE0885
      IF(ANORM.EQ.0.) GO TO 1183                                       MPSE0886
      DO 183 I=1,NOF                                                   MPSE0890
  183 DEFL(I)=DEFL(I)/XNORM                                            MPSE0900
 1183 WRITE(6,627) IMO,(DEFL(I),I=1,NOF)                               MPSE0910
  184 CONTINUE                                                         MPSE0920
  185 CONTINUE                                                         MPSE0930
      RETURN                                                           MPSE0940
  615 FORMAT(1H1,3X,32HEIGENVECTORS FOR MODES REQUESTED /)             MPSE0950
  618 FORMAT(1H1,3X,26HSTABILITY ANALYSIS RESULTS //)                  MPSE0960
  619 FORMAT(1H0,3X,13HGAIN FACTOR = 1PE11.4,1X,17HFOR SERVO ELEMENT I3)MPSE0970
  620 FORMAT(1H0,3X,13HGAIN FACTOR = 1PE11.4,1X,19HFOR CONTROL SURFACE1 MPSE0980
  622 FORMAT(1H0,3X,4HMODE,2X,12HEIGENVALUE-R,6X,12HEIGENVALUE-I,9X,   MPSE0990
     1 8HDAMPED,12X,8HUNDAMPED,8X,13HDAMPING RATIO,4X,7HTIME TO/       MPSE1000
     2 13X,4HMU-RPS,10X,9HOMEGA-RPS,6X,13HFREQUENCY-CPS,5X,            MPSE1010
     3 13HFREQUENCY-CPS,8X,4HZETA,9X,13H1/2 AMPLITUDE /)               MPSE1015
  623 FORMAT(1H ,3X,I3,1PE15.6,1P5E18.6)                               MPSE1020
  624 FORMAT(1H0,3X,20HEIGENVECTOR FOR MODE,I3                         MPSE1030
     1 //(1X,4(2X,1P2E15.7)))                                          MPSE1040
  625 FORMAT(1H ,3X,I3,1PE15.6,1P3E18.6,18X,1PE18.6)                   MPSE1050
  626 FORMAT(1H1,3X,58HSTRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR MPSE1060
     1EACH MODE /)                                                     MPSE1070
  627 FORMAT(1H0,3X,20HDEFLECTIONS FOR MODE,I3//(1X,4(2X,1P2E15.7)))   MPSE1080
      END                                                              MPSE1090
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      SUBROUTINE MPAS(A,B,STAB,VEC,C,BC,E,HH,PI,IANA,BINV,INDEX,SM,SPK,  MPAS0010
     1 SI,SMR,AMACH,APR,API,AMR,AMI,SAPR,SAPI,PSM,PSR,P,NII,NFR,NMODE,   MPAS0020
     2 NR,NM,INR,NA,MODE,MJ,MJI,GAMMA,NUF,DEFL,AMODE,NR,V)               MPAS0030
C     AERO-SERVO-ELASTIC ANALYSIS                                       MPAS0040
      COMMON/CONTRL/ MODE,NU,N1,N2,NTC,NA,NU,NG,NGS,K1,K2,NFR,NMM,NVEC,  MPAS0050
     1 NAB                                                              MPAS0060
      COMMON/INPUT/MU(5),AM(5),AK(10),DENS(5),SOS(5),GAIN(25)           MPAS0070
      COMMON/AERO/S,C,CBAR,S,UM,UM                                      MPAS0080
      DIMENSION A(NII,NII),B(NII,NII),STAB(MODE,7),VEC(MODE,NII),       MPAS0090
     1 SM(NM,NM),SMR(NA,NA),AMACH(1),APR(NFR,NMODE,NA),                 MPAS0100
     2 API(NFR,NMODE,NA),AMR(NFR,NMODE,NM),AMI(NFR,NMODE,NM),PSM(NM),   MPAS0110
     3 PSR(NA),P(MJ1),C(NII,NII),BC(NII,NII),E(NII),HH(NII),PI(NII),    MPAS0120
     4 IANA(NII),BINV(NII,1),INDEX(NII,3),SI(MJ1,MJ1),SMR(MJ1,MJ1),     MPAS0130
     5 SAPR(NFR,NMODE),SAPI(NFR,NMODE),DEFL(NUF),AMODE(NUF,NMODE)       MPAS0140
     6 ,NR(NII),V(1,1)                                                  MPAS0145
      COMPLEX VEC,E,DEFL,ANUM,V                                         MPAS0150
C                                                                       MPAS0160
C     SPLINE INTERPOLATION FOR MACH NO. AND REDUCED FREQUENCIES         MPAS0170
      CALL INTERP(AM,SM,SI,SMR,MINV,INDEX,NM,NM,1)                      MPAS0180
      CALL INTERP(AK,SPK,SI,SMR,MINV,INDEX,NK,NK,1)                     MPAS0190
C     DENSITY VARIATION                                                 MPAS0200
      DO 495 ID=1,ND                                                    MPAS0210
      NMACH=MU(ID)                                                      MPAS0220
      READ(5,503) (AMACH(IM),IM=1,NMACH)                                MPAS0230
C     MACH NUMBER VARIATION                                             MPAS0240
      DO 490 IM=1,NMACH                                                 MPAS0250
      NVEC=1                                                            MPAS0260
      IF(AMACH(IM).GE.0) NVEC=0                                         MPAS0270
      AMACH(IM)=ABS(AMACH(IM))                                          MPAS0280
      VEL=AMACH(IM)*SOS(ID)                                             MPAS0290
      CONAR=-.5*DENS(ID)*VEL**2*S/CBAR                                  MPAS0300
      CONAI=-.5*DENS(ID)*VEL*S                                          MPAS0310
      KAM=0                                                             MPAS0320
      IF(NM.GT.1) GO TO 290                                             MPAS0330
      KAM=1                                                             MPAS0340
      GO TO 310                                                         MPAS0350
  290 DO 300 I=1,NM                                                     MPAS0360
      XUM=ABS(AMACH(IM)-AM(I))                                          MPAS0370
      IF(XUM.LE.UM) GO TO 295                                           MPAS0380
      GO TO 300                                                         MPAS0390
  295 KAM=I                                                             MPAS0400
      GO TO 310                                                         MPAS0410
  300 CONTINUE                                                          MPAS0420
  310 IF(KAM.EQ.0) GO TO 350                                            MPAS0430
      REWIND 1                                                          MPAS0440
      DO 325 M=1,NM                                                     MPAS0450
      DO 325 K=1,NK                                                     MPAS0460
      IF(M.EQ.KAM) GO TO 320                                            MPAS0470
      READ(1)                                                           MPAS0480
      GO TO 325                                                         MPAS0490
  320 READ(1) ((APR(I,J,K),I=1,NFR),J=1,NMODE),((API(I,J,K),I=1,NFR),  MPAS0500
     1 J=1,NMODE)                                                       MPAS0510
      IF(K.EQ.NK) GO TO 400                                             MPAS0520
  325 CONTINUE                                                          MPAS0530
C     MACH NUMBER INTERPOLATION                                         MPAS0540
  350 CALL SPLINE(AMACH(IM),AM,SM,PSM,P,NM,NM,1)                        MPAS0550
      DO 390 K=1,NK                                                     MPAS0560
      REWIND 1                                                          MPAS0570
      DO 365 M=1,NM                                                     MPAS0580
      DO 365 KK=1,NK                                                    MPAS0590
      IF(KK.EQ.K) GO TO 360                                             MPAS0600
      READ(1)                                                           MPAS0610
      GO TO 365                                                         MPAS0620
  360 READ(1) ((AMR(I,J,M),I=1,NFR),J=1,NMODE),((AMI(I,J,M),I=1,NFR),  MPAS0630
     1 J=1,NMODE)                                                       MPAS0640
      IF(M.EQ.NM) GO TO 370                                             MPAS0650
  365 CONTINUE                                                          MPAS0660
  370 DO 380 I=1,NFR                                                    MPAS0670
      DO 380 J=1,NMODE                                                  MPAS0680
      APR(I,J,K)=0.0                                                    MPAS0690
      API(I,J,K)=0.0                                                    MPAS0700
      DO 380 M=1,NM                                                     MPAS0710
      APR(I,J,K)=APR(I,J,K)+PSM(M)*AMR(I,J,M)                           MPAS0720
  380 API(I,J,K)=API(I,J,K)+PSM(M)*AMI(I,J,M)                           MPAS0730
  390 CONTINUE                                                          MPAS0740
C     GAIN FACTOR OPTION                                                MPAS0750
  400 DO 485 IG=1,NG                                                    MPAS0760
      REWIND 2                                                          MPAS0770
      READ(2) ((A(I,J),I=1,NII),J=1,NII),((B(I,J),I=1,NII),J=1,NII)    MPAS0780
      IF(NGS) 420,425,401                                               MPAS0790
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  800 L=1                                                              MPAS0800
      II=NMODE+NOS                                                     MPAS0810
      IF(L-1) 815,810,805                                              MPAS0820
  805 JJ=NMODE+NC                                                      MPAS0830
      J=JC+JJ                                                          MPAS0840
      IF(GAIN(IG).EQ.0.) GO TO 808                                     MPAS0845
      A(II,JJ)=A(II,JJ)/GAIN(IG)                                       MPAS0850
      A(II,J)=A(II,J)/GAIN(IG)                                         MPAS0860
      B(II,J)=B(II,J)/GAIN(IG)                                         MPAS0870
      GO TO 823                                                        MPAS0880
  808 A1=A(II,JJ)                                                      MPAS0881
      A2=A(II,J)                                                       MPAS0882
      B1=B(II,J)                                                       MPAS0883
      DO 809 LL=1,NT                                                   MPAS0884
      A(II,LL)=0.                                                      MPAS0885
  809 B(II,LL)=0.                                                      MPAS0886
      A(II,JJ)=A1                                                      MPAS0887
      A(II,J)=A2                                                       MPAS0888
      B(II,J)=B1                                                       MPAS0889
      GO TO 823                                                        MPAS0890
  810 JJ=K2                                                            MPAS0895
      J=2*NC+JJ                                                        MPAS0900
      IF(GAIN(IG).EQ.0.) GO TO 813                                     MPAS0905
      A(II,J)=A(II,J)/GAIN(IG)                                         MPAS0910
      B(II,J)=B(II,J)/GAIN(IG)                                         MPAS0920
      GO TO 823                                                        MPAS0930
  813 A1=A(II,J)                                                       MPAS0931
      B1=B(II,J)                                                       MPAS0932
      DO 814 LL=1,NT                                                   MPAS0933
      A(II,LL)=0.                                                      MPAS0934
  814 B(II,LL)=0.                                                      MPAS0935
      A(II,J)=A1                                                       MPAS0936
      B(II,J)=B1                                                       MPAS0937
      GO TO 823                                                        MPAS0938
  815 JJ=K2                                                            MPAS0940
      J=2*NC+NT+JJ                                                     MPAS0950
      IF(GAIN(IG).EQ.0.) GO TO 818                                     MPAS0955
      B(II,J)=B(II,J)/GAIN(IG)                                         MPAS0960
      GO TO 823                                                        MPAS0970
  818 B1=B(II,J)                                                       MPAS0975
      DO 819 LL=1,NT                                                   MPAS0976
  819 B(II,LL)=0.                                                      MPAS0977
      B(II,J)=B1                                                       MPAS0978
      GO TO 823                                                        MPAS0979
  820 II=NP+IABS(NOS)                                                  MPAS0980
      JJ=NT+NP+IABS(NOS)                                               MPAS0990
      IF(GAIN(IG).EQ.0.) GO TO 821                                     MPAS0995
      A(II,II)=A(II,II)/GAIN(IG)                                       MPAS1000
      A(II,JJ)=A(II,JJ)/GAIN(IG)                                       MPAS1010
      B(II,JJ)=B(II,JJ)/GAIN(IG)                                       MPAS1020
      GO TO 823                                                        MPAS1021
  821 A1=A(II,II)                                                      MPAS1022
      A2=A(II,JJ)                                                      MPAS1023
      B1=B(II,JJ)                                                      MPAS1024
      DO 822 LL=1,NT                                                   MPAS1025
      A(II,LL)=0.                                                      MPAS1026
  822 B(II,LL)=0.                                                      MPAS1027
      A(II,II)=A1                                                      MPAS1028
      A(II,JJ)=A2                                                      MPAS1029
      B(II,JJ)=B1                                                      MPAS1030
  823 REWIND 3                                                         MPAS1035
      WRITE(3) ((A(I,J),I=1,NT),J=1,NT),((B(I,J),I=1,NT),J=1,NT)       MPAS1040
  825 ER=0.                                                            MPAS1050
      EI=0.                                                            MPAS1060
      IF=0                                                             MPAS1070
C     INTERPOLATION OF GENERALIZED AERO FORCES FOR EACH MODE           MPAS1080
  830 CALL SPLINE(KR,AK,SPR,PSR,P,NR,NR,1)                             MPAS1090
      DO 840 I=1,NPH                                                   MPAS1100
      DO 840 J=1,NMODE                                                 MPAS1110
      SAPR(I,J)=0.0                                                    MPAS1120
      SAPI(I,J)=0.0                                                    MPAS1130
      DO 835 K=1,NR                                                    MPAS1140
      SAPR(I,J)=SAPR(I,J)+PSR(K)*APR(I,J,K)                            MPAS1150
  835 SAPI(I,J)=SAPI(I,J)+PSR(K)*API(I,J,K)                            MPAS1160
  840 CONTINUE                                                         MPAS1170
      DO 845 I=1,NPH                                                   MPAS1180
      DO 845 J=1,NMODE                                                 MPAS1190
      JJ=NT+J                                                          MPAS1200
      A(I,JJ)=A(I,JJ)+CONAI*SAPI(I,J)                                  MPAS1210
      B(I,JJ)=B(I,JJ)+CONAR*SAPR(I,J)                                  MPAS1220
  845 CONTINUE                                                         MPAS1230
C     EIGENVALUE SOLUTION FOR EACH MODE                                MPAS1240
      CALL SOLV(ER,EI,ZETA,WA,A,B,C,BC,NR,NI,ZZ,IANA,BINV,INDEX,STAB,  MPAS1250
     1 VEC,NT,MODE,IMODE,NR,V)                                         MPAS1260
      IF(IE.EQ.IMODE) GO TO 850                                        MPAS1270
      ER=ER*OMP/VEL                                                    MPAS1280
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      IF(NOS.EQ.0) GO TO 448                                            MPAS1290
      NR=INO 3                                                          MPAS1300
      READ(3) ((A(I,J),I=1,NIT),J=1,NIT),((B(I,J),I=1,NIT),J=1,NIT)     MPAS1310
      GO TO 430                                                         MPAS1320
  448 NR=INO 2                                                          MPAS1330
      READ(2) ((A(I,J),I=1,NIT),J=1,NIT),((B(I,J),I=1,NIT),J=1,NIT)     MPAS1340
      GO TO 430                                                         MPAS1350
C     PRINT STABILITY RESULTS                                           MPAS1360
  450 WRITE(6,618)                                                      MPAS1370
      IF(NOS) 460,465,455                                               MPAS1380
  455 WRITE(6,619) GAIN(IG),NOS                                         MPAS1390
      GO TO 465                                                         MPAS1400
  460 INOS=IABS(NOS)                                                    MPAS1410
      WRITE(6,620) GAIN(IG),INOS                                        MPAS1420
  465 VELK=VEL*0.592484                                                 MPAS1430
      WRITE(6,621) SOS(IU),DENS(IU),AMACH(IM),VEL,VELK                  MPAS1440
      WRITE(6,622)                                                      MPAS1450
      DO 475 I=1,IMODE                                                  MPAS1460
      IF(STAB(I,2).EQ.0.) GO TO 470                                     MPAS1470
      WRITE(6,623) I,(STAB(I,J),J=1,7)                                  MPAS1480
      GO TO 475                                                         MPAS1490
  470 WRITE(6,625) I,(STAB(I,J),J=1,5),STAB(I,7)                        MPAS1500
  475 CONTINUE                                                          MPAS1510
      IF(NVEC.EQ.0) GO TO 485                                           MPAS1520
C     PRINT EIGENVECTORS                                                MPAS1530
      WRITE(6,615)                                                      MPAS1540
      DO 476 I=1,IMODE                                                  MPAS1550
  476 WRITE(6,624) I,(VEC(I,J),J=1,NIT)                                 MPAS1560
C     PRINT STRUCTURAL DEFLECTIONS                                      MPAS1570
      NR=INO 4                                                          MPAS1580
      IF(NPM.EQ.0) GO TO 477                                            MPAS1590
      READ(4)((AMODE(I,J),I=1,NOP),J=1,NPM)                             MPAS1600
  477 NP1=NPM+1                                                         MPAS1610
      IF(NMM.EQ.0) GO TO 478                                            MPAS1620
      READ(4)((AMODE(I,J),I=1,NOP),J=NP1,NPM)                           MPAS1630
  478 NPM1=NPMM+1                                                       MPAS1640
      READ(4)((AMODE(I,J),I=1,NOP),J=NPM1,NMODE)                        MPAS1650
      WRITE(6,626)                                                      MPAS1660
      DO 484 IMO=1,IMODE                                                MPAS1670
      DO 481 I=1,NOP                                                    MPAS1680
      DEFL(I)=0.0                                                       MPAS1690
      DO 480 J=1,NMODE                                                  MPAS1700
      K=NIT+J                                                           MPAS1710
  480 DEFL(I)=DEFL(I)+AMODE(I,J)*VEC(IMO,K)                             MPAS1720
  481 CONTINUE                                                          MPAS1730
      TS1=0.0                                                           MPAS1740
      DO 482 I=1,NOP                                                    MPAS1750
      ANORM=CABS(DEFL(I))                                               MPAS1760
      IF(TS1.GT.ANORM) GO TO 482                                        MPAS1770
      L=I                                                               MPAS1780
      TS1=ANORM                                                         MPAS1790
  482 CONTINUE                                                          MPAS1800
      ANORM=DEFL(L)                                                     MPAS1810
      ANORM=CABS(ANORM)                                                 MPAS1815
      IF(ANORM.EQ.0.) GO TO 1483                                        MPAS1816
      DO 483 I=1,NOP                                                    MPAS1820
  483 DEFL(I)=DEFL(I)/ANORM                                             MPAS1830
 1483 WRITE(6,627) IMO,(DEFL(I),I=1,NOP)                                MPAS1840
  484 CONTINUE                                                          MPAS1850
  485 CONTINUE                                                          MPAS1860
  490 CONTINUE                                                          MPAS1870
  495 CONTINUE                                                          MPAS1880
      RETURN                                                            MPAS1890
  503 FORMAT(8E12.0)                                                    MPAS1900
  615 FORMAT(1H1,3X,32HEIGENVECTORS FOR MODES REQUESTED /)              MPAS1910
  618 FORMAT(1H1,3X,26HSTABILITY ANALYSIS RESULTS //)                   MPAS1920
  619 FORMAT(1H0,3X,13HGAIN FACTOR = 1PE11.4,1X,17HFOR SERVO ELEMENT I3)MPAS1930
  620 FORMAT(1H0,3X,13HGAIN FACTOR = 1PE11.4,1X,19HFOR CONTROL SURFACE  MPAS1940
     1 I2)                                                              MPAS1950
  621 FORMAT(1H0,3X,18HSPEED OF SOUND = 1PE12.5,7H FT/SEC //4X,9HDENSITYMPAS1960
     1 = 1PE12.5,12H SLUGS/CU FT //4X,13HMACH NUMBER = 1PE12.5 //4X,    MPAS1970
     2 10HVELOCITY = 1PE12.5,8H FT/SEC, 1PE12.5,6H KNOTS //)            MPAS1980
  622 FORMAT(1H0,3X,4HMODE,2X,12HEIGENVALUE-R,6X,12HEIGENVALUE-I,9X,    MPAS1990
     1 8HDAMPED,12X,8HUNDAMPED,9X,7HREDUCED,8X,                         MPAS2000
     2 13HDAMPING RATIO,4X,7HTIME TO/13X,8HOMEG-RPS,10X,9HOMEGA-RPS,    MPAS2010
     3 8X,13HFREQUENCY-CPS,5X,13HFREQUENCY-CPS,8X,9HFREQUENCY,12X,      MPAS2020
     4 8HZETA,4X,13H1/2 AMPLITUDE /)                                    MPAS2021
  623 FORMAT(1H ,3X,I3,1PE15.5,1P6E18.5)                                MPAS2030
  624 FORMAT(1H0,3X,20HEIGENVECTOR FOR MODE,I3                          MPAS2040
     1 //(1X,8(2X,1PE15.7)))                                            MPAS2050
  625 FORMAT(1H ,3X,I3,1PE15.5,1P4E18.5,18X,1PE18.5)                    MPAS2060
  626 FORMAT(1H1,3X,58HSTRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR MPAS2070
     1EACH MODE /)                                                      MPAS2080
  627 FORMAT(1H0,3X,20HDEFLECTIONS FOR MODE,I3//(1X,8(2X,1PE15.7)))     MPAS2090
      END                                                               MPAS2100
```

```
      SUBROUTINE GENMAT(AM,GEN,TM,FRC,FMODE,RMODE,CMODE,MOF,MOR,NOF,NC,
     1 NFM,M1,M2)
C     GENMAT - FORM GENERALIZED MATRICES
      COMMON/CONTROL/MODE,N0,N1,N2,NTC,NA,ND,NG,NGS,K1,K2,NFM,NRM,NVEC,
     1 NAM
      DIMENSION AM(NOF,1),GEN(NFR,1),TM(M1,1),FRC(M2,1),FMODE(MOF,1),
     1 RMODE(MOR,1),CMODE(NOF,1)
C
      IF(NFM.EQ.0) GO TO 3
      CALL MATMPL(FMODE,AM,TM,MOF,NOF,M1,NFM,NOF,NOF,2)
      CALL MATMPL(TM,FMODE,FRC,M1,MOF,M2,NFM,NFM,NOF,1)
      DO 1 I=1,NFM
      DO 1 J=1,NFM
    1 GEN(I,J)=FRC(I,J)
      CALL MATMPL(TM,CMODE,FRC,M1,NOF,M2,NFM,NC,NOF,1)
      DO 2 I=1,NFM
      DO 2 J=1,NC
      JJ=NFR+J
    2 GEN(I,JJ)=FRC(I,J)
    3 IF(NRM.EQ.0) RETURN
      IF(NFM.EQ.0) GO TO 5
      CALL MATMPL(TM,RMODE,FRC,M1,MOR,M2,NFM,NRM,NOF,1)
      DO 4 I=1,NFM
      DO 4 J=1,NRM
      JJ=NFM+J
    4 GEN(I,JJ)=FRC(I,J)
    5 CALL MATMPL(RMODE,AM,TM,MOR,NOF,M1,NRM,NOF,NOF,2)
      CALL MATMPL(TM,RMODE,FRC,M1,MOR,M2,NRM,NRM,NOF,1)
      DO 6 I=1,NRM
      II=NFM+I
      DO 6 J=1,NRM
      JJ=NFM+J
    6 GEN(II,JJ)=FRC(I,J)
      CALL MATMPL(TM,CMODE,FRC,M1,NOF,M2,NRM,NC,NOF,1)
      DO 7 I=1,NRM
      II=NFM+I
      DO 7 J=1,NC
      JJ=NFM+J
    7 GEN(II,JJ)=FRC(I,J)
      IF(NFM.EQ.0) RETURN
      CALL MATMPL(TM,FMODE,FRC,M1,MOF,M2,NRM,NFM,NOF,1)
      DO 8 I=1,NRM
      II=NFM+I
      DO 8 J=1,NFM
    8 GEN(II,J)=FRC(I,J)
      RETURN
      END
```

```
      SUBROUTINE SOLV(A,B,P,GAMMA,[illegible],INV,INDEX,            SOLV0010
     1 STAB,[illegible])                                            SOLV0020
C     SOLV - EIGENVALUE SOLUTION AND STABILITY RESULTS              SOLV0030
      COMMON/CONTRL/ [illegible],NVEC,                              SOLV0040
     1 NAM                                                          SOLV0050
      DIMENSION A(NIT,1),B(NIT,1),C(NIT,1),BC(NIT,1),WR(1),WI(1),E(1),   SOLV0060
     1 IANA(1),INV(NIT,1),INDEX(NIT,3),STAB(MODE,1),VEC(MODE,1)     SOLV0070
     2 ,WK(1),V(1,1)                                                SOLV0075
      COMPLEX VEC,E,V                                               SOLV0080
C                                                                   SOLV0090
C     PRINT A AND B MATRICES                                        SOLV0100
      IF(NAM.EQ.0) GO TO 9                                          SOLV0110
      IF(KK.EQ.0) GO TO 9                                           SOLV0120
      WRITE(6,601)                                                  SOLV0130
      DO 5 I=1,NIT                                                  SOLV0140
    5 WRITE(6,602) (A(I,J),J=1,NIT)                                 SOLV0150
      WRITE(6,603)                                                  SOLV0160
      DO 6 I=1,NIT                                                  SOLV0170
    6 WRITE(6,602) (B(I,J),J=1,NIT)                                 SOLV0180
C     GENERATE DYNAMICAL MATRIX                                     SOLV0190
    9 DO 10 I=1,NIT                                                 SOLV0200
      DO 10 J=1,NIT                                                 SOLV0210
   10 BC(I,J)=GAMMA*A(I,J)+B(I,J)                                   SOLV0220
      CALL INVERS(NIT,BC,NIT,INV,0,DET,ISING,INDEX)                 SOLV0230
      IF(ISING.EQ.0) GO TO 15                                       SOLV0240
      WRITE(6,604)                                                  SOLV0250
      STOP                                                          SOLV0260
   15 CALL MATMPL(NC,BC,C,NIT,NIT,NIT,NIT,NIT,1)                    SOLV0270
C     EIGENVALUE SOLUTION FROM IMSL PACKAGE                         SOLV0280
      CALL EIGRF(C,NIT,NIT,0,E,V,NIT,WK,IER)                        SOLV0300
      DO 35 I=1,NIT                                                 SOLV0310
      IF(CABS(E(I)).EQ.0.0) GO TO 20                                SOLV0330
      P(I)=GAMMA-1./E(I)                                            SOLV0340
      GO TO 30                                                      SOLV0350
   20 E(I)=(-1.E30,0.)                                              SOLV0360
   30 WR(I)=REAL(E(I))                                              SOLV0370
      WI(I)=AIMAG(E(I))                                             SOLV0380
   35 CONTINUE                                                      SOLV0390
C     LINING UP THE REDUCED FREQUENCY DETERMINED BY THE EIGENVALUE  SOLV0400
      DO 45 I=1,NIT                                                 SOLV0410
      DO 40 J=I,NIT                                                 SOLV0420
      IF(WI(I).LE.WI(J)) GO TO 40                                   SOLV0430
      A1=WI(J)                                                      SOLV0440
      A2=WR(J)                                                      SOLV0450
      WI(J)=WI(I)                                                   SOLV0460
      WR(J)=WR(I)                                                   SOLV0470
      WI(I)=A1                                                      SOLV0480
      WR(I)=A2                                                      SOLV0490
   40 CONTINUE                                                      SOLV0500
   45 CONTINUE                                                      SOLV0510
      NP=0                                                          SOLV0520
      NI=0                                                          SOLV0530
      IM=0                                                          SOLV0540
      DO 60 I=1,NIT                                                 SOLV0550
      IF(WI(I).LT.0) GO TO 60                                       SOLV0560
      NP=NP+1                                                       SOLV0570
      IF(WI(I).NE.0.0) GO TO 55                                     SOLV0580
      IF(NNO.EQ.0) IOM=I                                            SOLV0590
      NI=NI+1                                                       SOLV0600
      NW=1                                                          SOLV0610
   60 CONTINUE                                                      SOLV0620
      IOE=IOM+NI-1                                                  SOLV0630
      DO 66 I=IOM,IOE                                               SOLV0640
```

```
      DO 67 J=I,100                                                    SOLV0650
      IF(ABS(RR(I)).LE.ABS(RR(J))) GO TO 67                            SOLV0660
      AR=RR(J)                                                         SOLV0670
      AI=RI(J)                                                         SOLV0680
      RR(J)=RR(I)                                                      SOLV0690
      RI(J)=RI(I)                                                      SOLV0700
      RR(I)=AR                                                         SOLV0710
      RI(I)=AI                                                         SOLV0720
   67 CONTINUE                                                         SOLV0730
   68 CONTINUE                                                         SOLV0740
      IF(MA.EQ.9) GO TO 80                                             SOLV0750
      DO 55 I=1,NIT                                                    SOLV0760
      IF(ER.EQ.0..AND.RR(I).EQ.0.) GO TO 50                            SOLV0770
      IF(RI(I).LE.0.) GO TO 55                                         SOLV0780
      NL=I                                                             SOLV0790
      GO TO 60                                                         SOLV0800
C     COMPUTE STABILITY RESULTS AND EIGENVECTORS FROM REAL ROOTS       SOLV0810
   50 IE=IE+1                                                          SOLV0820
      CALL RESULT(IE,RR(I),RI(I),NIT,NVEC,A,B,C,E,INDEX,IANA,RINV,STAB,SOLV0830
     1 VEC,MODE,ER)                                                    SOLV0840
   55 CONTINUE                                                         SOLV0850
   60 IF(ER.NE.0.) GO TO 65                                            SOLV0860
      E=RI(NL)                                                         SOLV0870
      RETURN                                                           SOLV0880
C     DETERMINE OMEGA FROM OSCILLATORY ROOTS                           SOLV0890
C     ESTIMATE OMEGA FOR REAL MODE                                     SOLV0900
   65 NN=NL+1                                                          SOLV0910
      IF(NN.GT.NIT) GO TO 78                                           SOLV0920
      TEST=ABS(RI(NL)-E)                                               SOLV0930
   70 TEST1=ABS(RI(NN)-E)                                              SOLV0940
      IF(TEST.LT.TEST1) GO TO 75                                       SOLV0950
      TEST=TEST1                                                       SOLV0960
      NN=NN+1                                                          SOLV0970
      IF(NN.GT.NIT) GO TO 78                                           SOLV0980
      GO TO 70                                                         SOLV0990
   75 E=RI(NN)                                                         SOLV1000
   78 NE=NN-1                                                          SOLV1010
      IE=IE+1                                                          SOLV1020
      IMODE=MODE                                                       SOLV1030
      IF(NN.GT.NIT) IMODE=IE                                           SOLV1040
C     COMPUTE STABILITY RESULTS AND EIGENVECTOR FOR MODE               SOLV1050
      CALL RESULT(IE,RR(NE),RI(NE),NIT,NVEC,A,B,C,E,INDEX,IANA,RINV,   SOLV1060
     1 STAB,VEC,MODE,ER)                                               SOLV1070
      RETURN                                                           SOLV1080
C     COMPUTE SERVO-ELASTIC RESULTS - NO AERO                          SOLV1090
   80 IMODE=MODE                                                       SOLV1100
      IF(NPR.LT.MODE) IMODE=NPR                                        SOLV1110
      DO 85 I=1,NIT                                                    SOLV1120
      IF(RI(I).LT.0.) GO TO 85                                         SOLV1130
      IE=IE+1                                                          SOLV1140
      CALL RESULT(IE,RR(I),RI(I),NIT,NVEC,A,B,C,E,INDEX,IANA,RINV,STAB,SOLV1150
     1 VEC,MODE,ER)                                                    SOLV1160
      IF(IE.EQ.IMODE) RETURN                                           SOLV1170
   85 CONTINUE                                                         SOLV1180
      RETURN                                                           SOLV1190
  901 FORMAT(1H1,2X,6HA MATRIX /)                                      SOLV1200
  902 FORMAT(1H /(1P8E16.7))                                           SOLV1210
  903 FORMAT(1H1,2X,6HB MATRIX /)                                      SOLV1220
  904 FORMAT(1H0,2X,29HDYNAMICAL MATRIX IS SINGULAR.)                  SOLV1230
      END                                                              SOLV1240
```

```
      SUBROUTINE RESULT(M,RR,RI,N,NV,A,B,U,C2,C3,L1,L2,STAB,VEC,MODE,ER)
C     RESULT - COMPUTES STABILITY RESULTS AND EIGENVECTORS FOR MODE
      DIMENSION A(N,1),B(N,1),U(N,N),C2(N),C3(N),L1(N),
     1 L2(N),STAB(MODE,1),VEC(MODE,1)
      COMPLEX GAMA,U,C2,C3,VEC,EIGEN
C
C     STABILITY RESULTS
      STAB(M,1)=RR
      STAB(M,2)=RI
      STAB(M,3)=RI/6.2831853
      STAB(M,4)=SQRT(RR**2+RI**2)/6.2831853
      STAB(M,5)=ER
      STAB(M,6)=-RR/SQRT(RR**2+RI**2)
      STAB(M,7)=0.69315/10/(-RR)
C     EIGENVECTORS
      IF(NV.EQ.0) RETURN
      GAMA=CMPLX(RR,RI)
      DO 10 I=1,N
      DO 10 J=1,N
   10 U(I,J)=GAMA*A(I,J)+B(I,J)
      EIGEN=(0.,0.)
      CALL EIGVCT(U,C2,EIGEN,C3,L1,L2,N)
      DO 15 I=1,N
   15 VEC(M,I)=C3(I)
      RETURN
      END
```

```
      SUBROUTINE INTERP(XMK,S,SI,SMK,BINV,INDEX,N,N1)
C     INTERP - GENERATES CONSTANT PORTION OF LINEAR SPLINE INTERPOLATION
      DIMENSION XMK(1),S(N1,1),SI(N1,1),SMK(N1,1),BINV(N1,1),INDEX(N1,3)
C
      DO 30 I=1,N1
      DO 30 J=I,N1
      IF(I.NE.1) GO TO 10
      SMK(I,J)=1.
      IF(J.EQ.1) SMK(I,J)=0.
      GO TO 25
   10 SMK(I,J)=ABS(XMK(I-1)-XMK(J-1))**3+ABS(XMK(I-1)+XMK(J-1))**3
   25 SMK(J,I)=SMK(I,J)
   30 CONTINUE
      DO 40 I=1,N1
      DO 40 J=1,N
      SI(I,J)=0.
      IF(I-J.EQ.1) SI(I,J)=1.
   40 CONTINUE
      CALL INVERS(N1,SMK,N1,BINV,0,DET,ISING,INDEX)
      IF(ISING.EQ.1) GO TO 45
      WRITE(6,60)
      STOP
   45 CALL MATMPL(SMK,SI,S,N1,N1,N1,N1,N,N1,1)
   60 FORMAT(1H0,2X,40HSPLINE INTERPOLATION MATRIX IS SINGULAR.)
      RETURN
      END
```

```
      SUBROUTINE SPLINE(CMK,XMK,S,PS,P,N,N1)
C     SPLINE - GENERATES SPLINE INTERPOLATION MATRIX
      DIMENSION XMK(1),S(N1,1),PS(1),P(1)
C
      P(1)=1.
      DO 10 I=1,N
      II=I+1
      P(II)=ABS(CMK-XMK(I))**3+ABS(CMK+XMK(I))**3
   10 CONTINUE
C     LAGRANGIAN COEFFICIENTS
      DO 20 I=1,N
      PS(I)=0.
      DO 15 J=1,N1
   15 PS(I)=PS(I)+P(J)*S(J,I)
   20 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE INVERS(NDIM,A,N,B,M,DETERM,ISING,INDEX)
C
C     ***** INVERSE OR LINEAR EQUATIONS SOLVER ***************************
C     ********************************************************************
C
C     NDIM IS THE ACTUAL SIZE OF A IN CALLING PROGRAM,
C             EG.  A(NDIM,NDIM)
C     A IS SQUARE MATRIX TO BE INVERTED.
C     N IS SIZE OF UPPER LEFT PORTION BEING INVERTED.          MINIMUM
C     B IS COLUMN OF CONSTANTS (OPTIONAL INPUT). SUPPLY SPACE B(NDIM,1)
C     M IS THE NUMBER OF COLUMNS OF CONSTANTS
C     DETERM RETURNS THE VALUE OF DETERMINANT IF NON-SINGULAR
C     ISING RETURNS,2 IF MATRIX A(N,N) IS SINGULAR
C                  ,1 IF MATRIX A(N,N) IS NON-SINGULAR
C     INVERSE RETURNS IN A
C     SOLUTION VECTORS RETURN IN B
C     INDEX IS WORKING STORAGE (N,3)
C     ********************************************************************
      DIMENSION  A(NDIM,1),  B(NDIM,1), INDEX(N,3)
      EQUIVALENCE (IROW,JROW), (ICOLUM,JCOLUM), (AMAX, T, SWAP)
C
C     INITIALIZE
C
      DETERM = 1.0E0
      DO 10 J=1,N
   10 INDEX(J,3) = 0
      DO 130 I=1,N
C
C     SEARCH FOR PIVOT
C
      AMAX = 0.0E0
      DO 40 J=1,N
      IF(INDEX(J,3) .EQ. 1) GO TO 40
      DO 30 K=1,N
      IF(INDEX(K,3) - 1) 20,30,190
   20 IF( ABS( A(J,K) ) .LE. AMAX) GO TO 30
      IROW = J
      ICOLUM = K
      AMAX = ABS( A(J,K) )
   30 CONTINUE
   40 CONTINUE
      IF(AMAX.EQ.0.0) GO TO 190
      INDEX(ICOLUM,3) = INDEX(ICOLUM,3) + 1
      INDEX(I,1) = IROW
      INDEX(I,2) = ICOLUM
C
C     INTERCHANGE ROWS TO PUT PIVOT ELEMENT ON DIAGONAL
C
      IF (IROW .EQ. ICOLUM) GO TO 70
      DETERM = -DETERM
      DO 50 L=1,N
      SWAP = A(IROW,L)
      A(IROW,L) = A(ICOLUM,L)
   50 A(ICOLUM,L) = SWAP
      IF(M .LE. 0) GO TO 70
      DO 60 L=1,M
      SWAP = B(IROW,L)
      B(IROW,L) = B(ICOLUM,L)
   60 B(ICOLUM,L) = SWAP
C
C     DIVIDE PIVOT ROW BY PIVOT ELEMENT
C
   70 PIVOT = A(ICOLUM,ICOLUM)
      DETERM = DETERM * PIVOT
      A(ICOLUM,ICOLUM) = 1.0E0
      DO 80 L=1,N
   80 A(ICOLUM,L) = A(ICOLUM,L) / PIVOT
      IF(M .LE. 0) GO TO 100
      DO 90 L=1,M
   90 B(ICOLUM,L) = B(ICOLUM,L) / PIVOT
C
C     REDUCE NON PIVOT ROWS
C
  100 DO 130 L1=1,N
      IF(L1 .EQ. ICOLUM) GO TO 130
      T = A(L1,ICOLUM)
      A(L1,ICOLUM) = 0.0E0
      DO 110 L=1,N
```

```
  110 A(L1,L) = A(L1,L) - A(ICOLUM,L) * T                      INVHS079
      IF(M .LE. 0) GO TO 130                                   INVHS080
      DO 120 L=1,M                                             INVHS081
  120 B(L1,L)=B(L1,L) - B(ICOLUM,L) * T                        INVHS082
  130 CONTINUE                                                 INVHS083
C                                                              INVHS084
C     INTERCHANGE COLUMNS                                      INVHS085
C                                                              INVHS086
      DO 150 I=1,N                                             INVHS087
      L = N + 1 - I                                            INVHS088
      IF(INDEX(L,1) .EQ. INDEX(L,2)) GO TO 150                 INVHS089
      JROW = INDEX(L,1)                                        INVHS090
      JCOLUM = INDEX(L,2)                                      INVHS091
      DO 140 K=1,N                                             INVHS092
      SWAP = A(K,JROW)                                         INVHS093
      A(K,JROW) = A(K,JCOLUM)                                  INVHS094
      A(K,JCOLUM) = SWAP                                       INVHS095
  140 CONTINUE                                                 INVHS096
  150 CONTINUE                                                 INVHS097
      DO 170 K=1,N                                             INVHS098
      IF(INDEX(K,3) .EQ. 1) GO TO 160                          INVHS099
      ISING = 2                                                INVHS100
      GO TO 180                                                INVHS101
  160 CONTINUE                                                 INVHS102
  170 CONTINUE                                                 INVHS103
      ISING = 1                                                INVHS104
  180 RETURN                                                   INVHS105
  190 ISING = 2                                                INVHS106
      RETURN                                                   INVHS107
      END                                                      INVHS108
```

```
      SUBROUTINE MATMPL(A,B,C,MA,MB,MC,M,N,L,IOP)              MATM0010
CMATMPL     MATRIX MULTIPLICATION (REAL AND TWO-DIMENSIONAL)   MATM0020
C                                                              MATM0030
C     ROW DIMENSION OF MATRIX A IN CALLING PROGRAM = MA        MATM0040
C     ROW DIMENSION OF MATRIX B IN CALLING PROGRAM = MB        MATM0050
C     ROW DIMENSION OF MATRIX C IN CALLING PROGRAM = MC        MATM0060
C     M = NO. OF ROWS IN PRODUCT MATRIX C                      MATM0075
C     N = NO. OF COLUMNS IN C                                  MATM0080
C     L = COMMON DIMENSION OF A AND B                          MATM0090
C     IOP = 1, A X B = C                                       MATM0100
C           2, A(TRANSPOSE) X B = C                            MATM0110
C           3, A X B(TRANSPOSE) = C                            MATM0120
C                                                              MATM0130
      DIMENSION A(MA,1),B(MB,1),C(MC,1)                        MATM0140
C                                                              MATM0150
      GO TO (100,200,300),IOP                                  MATM0160
  100 DO 175 I=1,M                                             MATM0170
      DO 150 J=1,N                                             MATM0180
      C(I,J)=0.0                                               MATM0190
      DO 125 K=1,L                                             MATM0200
      C(I,J)=C(I,J)+A(I,K)*B(K,J)                              MATM0210
  125 CONTINUE                                                 MATM0220
  150 CONTINUE                                                 MATM0230
  175 CONTINUE                                                 MATM0240
      GO TO 400                                                MATM0250
  200 DO 275 I=1,M                                             MATM0260
      DO 250 J=1,N                                             MATM0270
      C(I,J)=0.0                                               MATM0280
      DO 225 K=1,L                                             MATM0290
      C(I,J)=C(I,J)+A(K,I)*B(K,J)                              MATM0300
  225 CONTINUE                                                 MATM0310
  250 CONTINUE                                                 MATM0320
  275 CONTINUE                                                 MATM0330
      GO TO 400                                                MATM0340
  300 DO 375 I=1,M                                             MATM0350
      DO 350 J=1,N                                             MATM0360
      C(I,J)=0.0                                               MATM0370
      DO 325 K=1,L                                             MATM0380
      C(I,J)=C(I,J)+A(I,K)*B(J,K)                              MATM0390
  325 CONTINUE                                                 MATM0400
  350 CONTINUE                                                 MATM0410
  375 CONTINUE                                                 MATM0420
  400 RETURN                                                   MATM0430
      END                                                      MATM0440
```

```
      SUBROUTINE EGNVCT ( C1, C2, EIGEN, C3, N1, N2, N)
C
C     SUBROUTINE TO OBTAIN EIGENVECTOR FROM REAL NON-SYMMETRIC
C     MATRICES FOR WHICH THE EIGENVALUE IS KNOWN.  THE METHOD
C     USED IS THE DIRECT METHOD OUTLINED IN ERR-F-   BY DR.
C     B. N. CUNNINGHAM
C
      COMPLEX C1(N,N), C2(N), C3(N),
     1         EIGEN, D1, D2, D3, D4, D5, D6, D8
      INTEGER N1(N),N2(N)
C
      II3 = N
      II2 = N - 1
      X1 = 0.0
      DO 20 J=1,N
      N1(J) = J
      N2(J) = J
      C1(J,J) = C1(J,J) - EIGEN
      DO 10 I=1,N
      X2 = CABS(C1(I,J))
      IF(X1-X2) 5,10,10
    5 X1 = X2
      I1 = I
      J1 = J
   10 CONTINUE
   20 CONTINUE
      DO 150 K6=2,N
      IF(CABS(C1(I1,J1)))50,30,50
   30 K5 = K6 - 1
C     SINGULAR MATRIX RETURN ZERO
      DO 36 I=1,N
   36 C3(I) = 0.0
      GO TO 1000
C
   50 D1 =(1.0,0.0)/C1(I1,J1)
      D2 = C1(I1,II3)
      D3 = C1(II3,J1)
      D4 = C1(II3,II3)
      DO 60 I=1,II2
      C3(I) = C1(I,J1)
      C1(I,J1) = C1(I,II3)
      C1(I,II3) = -C3(I)*D1
      D5 = -C1(I1,I)*D1
      C1(I1,I) = C1(II3,I)
      C1(II3,I) = D5
   60 CONTINUE
      C3(I1) = D3
      C1(I1,J1) = D4
      C1(II3,J1) = -D2*D1
      C1(I1,II3) = -D3*D1
      C1 (II3,II3) = D1
      IF(II3 .EQ. N) GO TO 80
      II4 = II3 + 1
      DO 70 I=II4,N
      D6 = C1(I1,I)
      C1 (I1,I) = C1(II3,I)
      C1(II3,I) = D6
      C3(I) = C1(I,J1)
      C1(I,J1) = C1(I,II3)
   70 C1(I,II3) = C3(I)
   80 I=N1(J1)
      N1(J1) = N1(II3)
      N1(II3) = I
      I = N2(I1)
      N2(I1) = N2(II3)
      N2(II3) = I
      X1 = 0.0
      DO 140 J=1,II2
      D8 = C1(II3,J)
      DO 130 I=1,II2
      C1(I,J) = C1(I,J) + C3(I)*D8
      X2 = CABS(C1(I,J))
      IF(X1-X2) 120,130,130
```

```
  120 X1 = X2
      I1 = I
      J1 = J
  130 CONTINUE
  140 CONTINUE
      II3 = II3 - 1
      II2 = II2 - 1
  150 CONTINUE
C
  160 C3(2) = C1(2,1)
      C3(1) = (1.0,0.0)
      DO 180 J=3,N
      C3(J) = (0.0,0.0)
      J1 = J-1
      DO 170 I=1,J1
      C3(J) = C3(J) + C3(I)*C1(J,I)
  170 CONTINUE
  180 CONTINUE
      IF(CABS(C1(1,1) ) .LT. 1.0E-20) GO TO 202
      DO 201 KK=1,2
C
      DO 184 J=1,N
      I1 = N2(J)
      DO 182 I=1,N
      IF( I1 .EQ. N1(I) ) GO TO 184
  182 CONTINUE
  184 C2(J) = C3(I)
C
      DO 190 J=2,N
      I1 = N - J + 1
      J1 = I1 + 1
      DO 185 I=1,I1
      C2(I) = C2(I) + C1(I,J1)*C2(J1)
  185 CONTINUE
  190 CONTINUE
      D1 = C1(1,1)/C2(1)
      C3(1) = (1.0,0.0)
      DO 200 J=2,N
      I1 = J - 1
      C3(J) = C2(J) - C1(J,J)*D1
      DO 195 I=1,I1
      C3(J) = C3(J) + C1(J,I)*C3(I)
  195 CONTINUE
  200 CONTINUE
  201 CONTINUE
C
C     C3(I) NOW CONTAINS THE EIGENVECTOR WHICH MUST BE RE-ARRANGED
C     ACCORDING TO THE ORDER DICTATED BY N1(I) BACK TO THE ORIGINAL
C     ORDER.
C
  202 DO 230 I=1,N
      I1 = N1(I)
      N1(I) = I
  205 IF(I1-I) 210,230,210
  210 D1 = C3(I1)
      C3(I1) = C3(I)
      C3(I) = D1
      K = N1(I1)
      N1(I1) = I1
      I1 = K
      GO TO 205
  230 CONTINUE
  240 N1(1) = 2
C
 1000 RETURN
      END
```

APPENDIX A

THEORETICAL DERIVATION FROM REFERENCE 1

2.0 THE CLOSED-LOOP AERO-SERVO-ELASTIC STABILITY PROBLEM

2.1 NOMENCLATURE

A	Element of coefficient matrix of $\{\dot{v}\}$ in Equati - (2-4)
AIC	Aerodynamic influence coefficient
a	Amplitude of generalized displacement coordinate
B	Element of coefficient matrix of $\{v\}$ in Equation (2-4)
b_r	Reference dimension
C	Element of discrete damping matrix
c	Element of generalized damping matrix
D	Element of differentiation matrix
e	Servo signal
F	Element of forcing function matrix
g_F	Modal structural damping coefficient
H	Element of interpolation matrix
h	Deflection of aeroelastic system
h_δ	Deflection due to unit shaft rotation
I	Element of unit matrix
i	Imaginary unit
K	Servo system gain constant; element of stiffness matrix
$\mathfrak{k}$	Element of generalized stiffness matrix
k	Reduced frequency (Strouhal number)
M	Element of discrete mass matrix
m	Element of generalized mass matrix
Q	Element of generalized force matrix
Q_a	Element of AIC matrix

Re( )	Denotes real part of ( )
s	Laplace transform parameter
T	Servo system time constant
u	$u = \dot{x}$
V	Amplitude of v; velocity of flight
v	$\{v\} = \lfloor u \ x \ y \ z \rfloor^T$
X	Element of coefficient matrix of second order variables
x	Second order variable; forward Cartesian coordinate
Y	Element of coefficient matrix of first order variables
y	First order variable
Z	Element of coefficient matrix of zero order variables
z	Zero order variable
γ	Eigenvalue, i. e., coefficient in Equation (2-5) to describe the time dependence of transient motion. Note $Re(\gamma) = 0$ indicates neutral stability.
γ_o	Shift value of γ in eigenvalue problem
δ	Shaft rotation of control surface
ζ	Ratio of viscous damping coefficient to the critical viscous damping coefficient
λ	Eigenvalue of Equation (2-8)
λ_o	Shift value of λ in eigenvalue problem
ω	Angular frequency
$(\bar{\ })$	Denotes complex amplitude

<u>Subscripts</u>

a	Actuator; aerodynamic
F	Flexible body motion
g	Gyro
h	Hinge line

p	Potentiometer
R	Rigid body motion
x	Corresponds to magnitude of second order variable
$\dot{x}$	Corresponds to velocity of second order variable
$\ddot{x}$	Corresponds to acceleration of second order variable
y	Corresponds to magnitude of first order variable
$\dot{y}$	Corresponds to velocity of first order variable
z	Corresponds to magnitude of zero order variable
δ	Control surface rotation

Matrix Notation

$[\ ]$	Square or rectangular
$[\ ]^T$	Transpose
$[\ ]^{-1}$	Inverse
$\{\ \}$	Column
$\lfloor\ \rfloor$	Row
$\lceil\ \rfloor$	Diagonal

2.2 THEORETICAL DERIVATION

All components of an aero-servo-elastic system can be regarded as being composed of elements whose characteristics are described by second order (or lower) differential equations. Let $\{x\}$ denote the set of second order variables, $\{y\}$ the set of first order variables, and $\{z\}$ the set of zero order variables. The aero-servo-elastic equations of motion can be written in the general form

$$
\begin{aligned}
&[X_{\ddot{x}}]\{\ddot{x}\} + [X_{\dot{x}}]\{\dot{x}\} + [X_x]\{x\} \\
&+ [Y_{\dot{y}}]\{\dot{y}\} + [Y_y]\{y\} + [Z_z]\{z\} = \{F\}
\end{aligned}
\tag{2-1}
$$

where $\{F\}$ denotes a forcing function. The mathematical formulation of the stability problem requires first order differential equations. We introduce the variable

$$
\{u\} = \{\dot{x}\} \tag{2-2}
$$

and combine Equations (2-1) and (2-2) into the matrix form

$$
\begin{aligned}
&\begin{bmatrix} X_{\ddot{x}} & X_{\dot{x}} & Y_{\dot{y}} & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{u} \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{Bmatrix} \\
&+ \begin{bmatrix} 0 & X_x & Y_y & Z_z \\ -I & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u \\ x \\ y \\ z \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}
\end{aligned}
\tag{2-3}
$$

which we may abbreviate as

$$
[A]\{\dot{v}\} + [B]\{v\} = \{Q\} \tag{2-4}
$$

By setting the forcing function to zero, and letting

$$\{v\} = \{V\} \exp(\gamma t) \tag{2-5}$$

we obtain the eigenvalue formulation of the stability problem

$$(\gamma[A] + [B]) \{V\} = 0 \tag{2-6}$$

Instability occurs when the velocity and/or system gains are such that the real part of γ is positive.

Since $[A]$ is singular and $[B]$ may be singular, we let

$$\gamma = \gamma_o - 1/(\lambda - \lambda_o) \tag{2-7}$$

where γ_o and λ_o are arbitrarily chosen complex numbers. Then Equation (2-6) can be rewritten in the canonical form of the eigenvalue problem as

$$\lambda\{V\} = (\gamma_o[A] + [B])^{-1} \left[[A] + \lambda_o (\gamma_o[A] + [B])\right]\{V\} \tag{2-8}$$

where the new eigenvalue is

$$\lambda = \lambda_o - 1/(\gamma - \gamma_o) \tag{2-9}$$

The eigenvalue extraction leads to convergence to the eigenvalues λ corresponding to the value of γ closest to γ_o. The non-Hermitian matrices yield roots that appear separately, as close pairs, or as complex conjugates.

The frequency and damping for each mode are found from the real and imaginary parts of each eigenvalue. Letting α denote the decay rate coefficient, β the damped frequency, ζ the viscous damping coefficient, and ω the undamped frequency of each mode, we may write

$$\gamma = \alpha + i\beta \tag{2-10a}$$

$$= \omega\left(\zeta + i\sqrt{1 - \zeta^2}\right) \tag{2-10b}$$

from which we find

$$\omega = \sqrt{\alpha^2 + \beta^2} \tag{2-11}$$

and

$$\zeta = a/\omega \tag{2-12}$$

The equations of motion of the aeroelastic system including the shaft rotation of the control surface (flipper) appear as

$$[M]\{\ddot{h}\} + [C]\{\dot{h}\} + [K]\left(\{h\} - [h_\delta]\{\delta\}\right) = \{F\} \tag{2-13}$$

where $\{h\}$ denotes the displacement of the control point masses in the lumped-parameter aeroelastic system, $[M]$ is the mass matrix, $[C]$ is the viscous damping matrix, $[K]$ is the stiffness matrix, $[h_\delta]$ is the flipper displacement matrix (the elements of $[h_\delta]$ are zero for points off the flipper and equal $(x_h - x_i)$ for the $i^{\underline{th}}$ control point where x_h is the hinge line coordinate), and $\{F\}$ is the external force matrix. For the purposes of closed-loop stability analysis, the external force of interest is the aerodynamic force induced by the motion. A survey of unsteady aerodynamic influence coefficients (AICs) has been given in Reference 3. For present purposes it is sufficient to write the aerodynamic force as

$$\{F\} = -[\overline{Q}_a]\{\ddot{h}\} \tag{2-14}$$

where $[\overline{Q}_a]$ is a <u>complex</u> matrix of oscillatory AICs <u>valid only for harmonic motion</u>. The AICs $[\overline{Q}_a]$ are dependent on the planform, the altitude, the flight Mach number, and the reduced frequency (Strouhal number) of the motion $k = \omega b_r/V$ where ω is the frequency, b_r is a reference dimension, and V is the velocity of flight. This limitation to harmonic motion reflects the state-of-the-art of unsteady aerodynamic theory, viz., considerably more solutions have been found for harmonic motion than for arbitrary transient motion. For this reason flutter analysis has traditionally required a trial-and-error solution to find the velocity and frequency for neutrally stable oscillations. To the same extent the aero-servo-elastic stability analysis must be carried out by trial and error. Equations (2-13) and (2-14) may be combined to appear as

$$[\overline{M}]\{\ddot{h}\} + [C]\{\dot{h}\} + [K]\left(\{h\} - [h_\delta]\{\delta\}\right) = 0 \tag{2-15}$$

where the mass matrix now includes the aerodynamics.

$$[\overline{M}] = [M] + [\overline{Q}_a] \tag{2-16}$$

The inputs to the hydraulic actuators come from rate gyro and accelerometer feedback loops. The angular velocity of a rate gyro may be found by numerical differentiation of the displacement velocities.

$$\dot{h}' = \lfloor D \rfloor \{\dot{h}\} \tag{2-17}$$

where $\lfloor D \rfloor$ is a differentiation row matrix. Methods of numerical differentiation have been discussed thoroughly, e. g., by Milne[4]. If the differentiation is carried out locally, i. e., "in-the-small", then the elements of $\lfloor D \rfloor$ will only be nonzero for control points surrounding the gyro location, e. g., parabolic differentiation would involve only the three control points closest to the gyro. The acceleration at an accelerometer is found by numerical interpolation of the displacement accelerations.

$$\ddot{h} = \lfloor H \rfloor \{\ddot{h}\} \tag{2-18}$$

where $\lfloor H \rfloor$ is an interpolation row matrix. Methods of numerical interpolation have also been discussed by Milne[4]. If the interpolation is also carried out locally, the elements of $\lfloor H \rfloor$ will also only be nonzero for control points surrounding the accelerometer location.

Since the aeroelastic system has a large number of degrees of freedom, it is desirable to reduce them by a series or modal method. We write the deflections in terms of a series of vibration modes, rigid-body modes, and the shaft rotation(s) of the flipper(s).

$$\{h\} = [h_F]\{a_F\} + [h_R]\{a_R\} + [h_\delta]\{\delta\} \tag{2-19}$$

where $[h_F]$ is a matrix of restrained and/or free-free vibration modes, $\{a_F\}$ is the corresponding set of generalized coordinates of the vibration modes, $[h_R]$ is the matrix of rigid body deflection modes, and $\{a_R\}$ is the set of amplitudes of the rigid body motions. Substituting Equation (2-19) into Equation (2-15) leads to a truncation error in the solution since the series, Equation (2-19), is finite. The error matrix may be interpreted physically as a distributed force on the system. If we impose the condition that this error does no work in any of the flexible or rigid modes (N. B., this is the method of Galerkin) then the generalized equations of motion are found by premultiplying Equation (2-15) by $[h_F]^T$ and setting

to zero, and also by premultiplying by $\lceil h_R \rceil^T$ and setting to zero. Substituting Equation (2-19) into Equation (2-15) and premultiplying by $\lceil h_F \rceil^T$ leads to

$$
\begin{aligned}
&\lceil \bar{m}_F \rceil \{\ddot{a}_F\} + \lceil \bar{m}_{FR} \rceil \{\ddot{a}_R\} + \lceil \bar{m}_{F\delta} \rceil \{\ddot{\delta}\} \\
&\quad + \lceil c_F \rceil \{\dot{a}_F\} + \lceil k_F \rfloor \{a_F\} = 0
\end{aligned}
\tag{2-20}
$$

where we have noted that rigid body displacements produce no internal damping or structural forces, and

$$\lceil \bar{m}_F \rceil = \lceil h_F \rceil^T \lceil \bar{M} \rceil \lceil h_F \rceil \tag{2-21}$$

$$\lceil \bar{m}_{FR} \rceil = \lceil h_F \rceil^T \lceil \bar{M} \rceil \lceil h_R \rceil \tag{2-22}$$

$$\lceil \bar{m}_{F\delta} \rceil = \lceil h_F \rceil^T \lceil \bar{M} \rceil \lceil h_\delta \rceil \tag{2-23}$$

$$\lceil c_F \rceil = \lceil h_F \rceil^T \lceil C \rceil \lceil h_F \rceil \tag{2-24}$$

and

$$\lceil k_F \rfloor = \lceil h_F \rceil^T \lceil K \rceil \lceil h_F \rceil \tag{2-25}$$

We also have noted that the generalized stiffness matrix has a diagonal form by virtue of the orthogonality of vibration modes, whether they be restrained modes or free-free modes,* and can be written in terms of the generalized mass matrix and frequency matrix as

$$\lceil k_F \rfloor = \lceil \omega_F^2 \rfloor \lceil m_F \rfloor \tag{2-26a}$$

$$= \lceil \omega_F^2 \, m_F \rfloor \tag{2-26b}$$

where

$$\lceil m_F \rfloor = \lceil h_F \rceil^T \lceil M \rceil \lceil h_F \rceil \text{ , diagonal elements} \tag{2-27}$$

---

*This is also the case for arbitrarily chosen modes, e. g., modes given by polynomial expressions, if their associated frequencies can be defined. However, inclusion of arbitrary modes leads to a convergence requirement of a large number of terms in Equation (2-19) and therefore will not be permitted in this formulation.

and ω_F is the frequency of the vibration mode $\{h_F\}$. The generalized damping matrix does not have a diagonal form, as does the generalized stiffness, but may be assumed so as an approximate means of including structural damping. If we represent the structural damping in its equivalent viscous form we may write

$$\lceil c_F \rfloor = \lceil k_F g_F / \omega_F \rfloor \tag{2-28a}$$

$$= \lceil g_F \omega_F m_F \rfloor \tag{2-28b}$$

where g_F is the structural damping coefficient (N. B., $g_F = 2\zeta_F$ where ζ is the ratio of viscous damping coefficient to the critical viscous damping coefficient) corresponding to the vibration mode $\{h_F\}$.

Substituting Equation (2-19) into Equation (2-15) and premultiplying by $[h_R]^T$ leads to

$$[\bar{m}_{RF}]\{\ddot{a}_F\} + [\bar{m}_R]\{\ddot{a}_R\} + [\bar{m}_{R\delta}]\{\ddot{\delta}\} = 0 \tag{2-29}$$

where we again have noted that rigid body displacements produce no internal damping or structural forces, and

$$[\bar{m}_{RF}] = [h_R]^T[\bar{M}][h_F] \tag{2-30}$$

$$[\bar{m}_R] = [h_R]^T[\bar{M}][h_R] \tag{2-31}$$

and $$[\bar{m}_{R\delta}] = [h_R]^T[\bar{M}][h_\delta] \tag{2-32}$$

If the generalized aerodynamic forces for specific modes are considered, we may rewrite Equations (2-21)-(23) and (30)-(32) as

$$[\bar{m}_F] = \lceil m_F \rfloor + [\bar{Q}_F] \tag{2-33}$$

$$[\bar{m}_{FR}] = [m_{FR}] + [\bar{Q}_{FR}] \tag{2-34}$$

$$[\bar{m}_{F\delta}] = [m_{F\delta}] + [\bar{Q}_{F\delta}] \tag{2-35}$$

$$[\bar{m}_R] = [m_R] + [\bar{Q}_R] \tag{2-36}$$

$$[\bar{m}_{RF}] = [m_{FR}]^T + [\bar{Q}_{RF}] \tag{2-37}$$

$$[\bar{m}_{R\delta}] = [m_{R\delta}] + [\bar{Q}_{R\delta}] \tag{2-38}$$

where

$$[m_{FR}] = [h_F]^T [M][h_R] \tag{2-39}$$

$$[m_{F\delta}] = [h_F]^T [M][h_\delta] \tag{2-40}$$

$$[m_R] = [h_R]^T [M][h_R] \tag{2-41}$$

and

$$[m_{R\delta}] = [h_R]^T [M][h_\delta] \tag{2-42}$$

Note that $[m_{FR}]$ vanishes if free-free modes are used throughout. Also, $[m_R]$ is the rigid body mass matrix consisting of total mass, static unbalances about the coordinate origin (not necessarily the centroid), and moments and products of inertia about the coordinate origin. The generalized aerodynamic forces are found (e. g., from Volumes I or II of this report) for the appropriate vibration, rigid body, and flipper modes, including their coupling.

The matrix partitions in Equation (2-3) may now be defined as follows:

$$[X_{\ddot{x}}]\{\ddot{x}\} = \begin{array}{c c} & \begin{array}{cccc} m & n & c & s \end{array} \\ \begin{array}{c} m \\ n \\ c \\ ns \end{array} & \begin{bmatrix} [\bar{m}_F] & [\bar{m}_{FR}] & [\bar{m}_{F\delta}] & 0 \\ [\bar{m}_{RF}] & [\bar{m}_R] & [\bar{m}_{R\delta}] & 0 \\ 0 & 0 & [CC] & [CS2] \\ [FSA] & [RSA] & [SC] & [SS2] \end{bmatrix} \end{array} \begin{Bmatrix} \ddot{a}_F \\ \ddot{a}_R \\ \ddot{\delta} \\ \ddot{e}2 \end{Bmatrix} \tag{2-43}$$

$$[X_{\dot{x}}]\{\dot{x}\} = \begin{array}{cc} & \begin{array}{cccc} m & n & c & s \end{array} \\ \begin{array}{c} m \\ n \\ c \\ ns \end{array} & \begin{bmatrix} \lceil c_F \rfloor & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \lceil CC \rfloor & [CS2] \\ \lfloor FSG \rfloor & \lfloor RSG \rfloor & [SC] & [SS2] \end{bmatrix} \end{array} \begin{Bmatrix} \dot{a}_F \\ \dot{a}_R \\ \dot{\delta} \\ \dot{e}2 \end{Bmatrix} \qquad (2\text{-}44)$$

$$[X_x]\{x\} = \begin{array}{cc} & \begin{array}{cccc} m & n & c & s \end{array} \\ \begin{array}{c} m \\ n \\ c \\ ns \end{array} & \begin{bmatrix} \lceil k_F \rfloor & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \lceil CC \rfloor & [CS2] \\ 0 & 0 & [SC] & [SS2] \end{bmatrix} \end{array} \begin{Bmatrix} a_F \\ a_R \\ \delta \\ e2 \end{Bmatrix} \qquad (2\text{-}45)$$

where e2 refers to the set of second order servo elements and

CC = Control surface coefficients.

CS2 = Coefficients for second order servo element from which there is input to a control surface.

FSA = Elements of the matrix $-K_a \lfloor H \rfloor \lceil h_F \rfloor$ for the accelerometer into which there is input from the body acceleration.

FSG = Elements of the matrix $-K_g \lfloor D \rfloor \lceil h_F \rfloor$ for the rate gyro into which there is input from the body angular rate.

RSA = Elements of the matrix $-K_a \lfloor H \rfloor \lceil h_R \rfloor$ for the accelerometer into which there is input from the body acceleration.

RSG = Elements of the matrix $-K_g \lfloor D \rfloor \lceil h_R \rfloor$ for the rate gyro into which there is input from the body angular rate.

SC = Coefficients for control surface from which there is input to a servo element.

SS2 = Coefficients for second order servo element from which there is input to another servo element.

The number of rows and columns in each partition is indicated by

m = Number of flexible modes.

n = Number of rigid body modes.

c = Number of control surfaces.

ns = Total number of servo elements (zero, first and second order), ns = s + q + r.

s = Number of second order servo elements.

q = Number of first order servo elements.

r = Number of zero order servo elements.

$$[Y_{\dot{y}}]\{\dot{y}\} = \begin{array}{c c} & q \\ \begin{array}{c} m \\ n \\ c \\ ns \end{array} & \begin{bmatrix} 0 \\ 0 \\ [CS1] \\ [SS1] \end{bmatrix} \end{array} \{\dot{e}1\} \tag{2-46}$$

$$[Y_{y}]\{y\} = \begin{array}{c c} & q \\ \begin{array}{c} m \\ n \\ c \\ ns \end{array} & \begin{bmatrix} 0 \\ 0 \\ [CS1] \\ [SS1] \end{bmatrix} \end{array} \{e1\} \tag{2-47}$$

where e1 refers to the set of first order servo elements and

CS1 = Coefficients for first order servo element from which there is input to a control surface.

SS1 = Coefficients for first order servo element from which there is input to another servo element.

$$
[Z_z]\{z\} = \begin{array}{c c}
 & r \\
\begin{array}{c} m \\ n \\ c \\ ns \end{array} & \begin{bmatrix} 0 \\ 0 \\ [CS0] \\ [SS0] \end{bmatrix}
\end{array} \{e0\} \tag{2-48}
$$

where e0 refers to the set of zero order servo elements and

CS0 = Coefficients for zero order servo element from which there is input to a control surface.

SS0 = Coefficients for zero order servo element from which there is input to another servo element.

A study of the format of Equations (2-43)-(48) shows that each of the six matrices is composed of partitions that may be described as aeroelastic, servoelastic, or simply servo terms. PASES, a computer program designed to calculate closed-loop aero-servo-elastic stability, forms the various matrices from the servo partitions which are input directly and the aeroelastic partitions which are generated from basic input (e. g., modes, frequencies, mass, and aerodynamic data).

APPENDIX B

AN APPLICATION OF PROGRAM MPASES
TO A
TYPICAL AIR-TO-AIR MISSILE

Introduction

The computer program MPASES developed in this report used the same example problem to demonstrate its usage as had been used in the original version of the program. That example of an air-to-air missile was not a very practical one to the extent that it had no wing, the weight of the aft end of the fuselage was assumed to oscillate with the control surface (flipper), and there was only a single (rate-gyro) feedback loop. In addition, the example used an incorrect transfer function for the actuator [Eq. (78) should have read $\delta/e_3 = K_a/s(T_a s + 1)$].

The new example studied here is still somewhat idealized but is more practical to the extent that a wing, with its attendant aerodynamic loads, is added, and a light-weight dynamically balanced flipper with bending and torsion degrees of freedom is considered separately from the body. A more realistic servo system is also considered with three feedback loops, a rate feedback loop (with a rate-gyro as before), and attitude and acceleration feedback loops.

In addition to being a more practical problem which demonstrates a more general use of Program MPASES, the sample also illustrates certain peripheral aspects of the problem. One is the calculation of a coupled mass matrix for the flipper. Another is the calculation of the wing and

flipper aerodynamic influence coefficients from Piston Theory for a more practical range of supersonic Mach numbers and their assembly to obtain the proper partitioned format for the complete vehicle.

The servoelastic stability problem is considered first, and then the aerodynamic influence coefficients are added next to consider the aero-servo-elastic stability analysis.

Typical Air-to-Air Missile Example

The typical air-to-air missile is idealized as shown in Fig. 3. The fuselage is idealized as a uniform beam with ten equally spaced masses. The wing is assumed to be rectangular and massless, rigid in the spanwise direction and to bend with the fuselage in the chordwise direction. The flipper is also rectangular and is assumed to be rigid in both spanwise and chordwise directions but to have root flexibility in bending and torsion. The flipper mass distribution is simulated by eight masses connected by rigid massless bars to the four degrees of freedom of the flipper. The coupled mass matrix for the flipper is obtained from the least squares solution in Appendix B of Ref. 12. The various input matrices to MPASES are derived below.

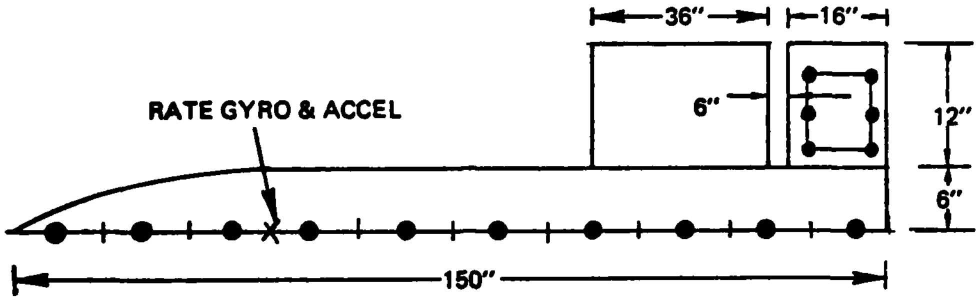

Fig. 3 - Idealization of Typical Air-to-Air Missile

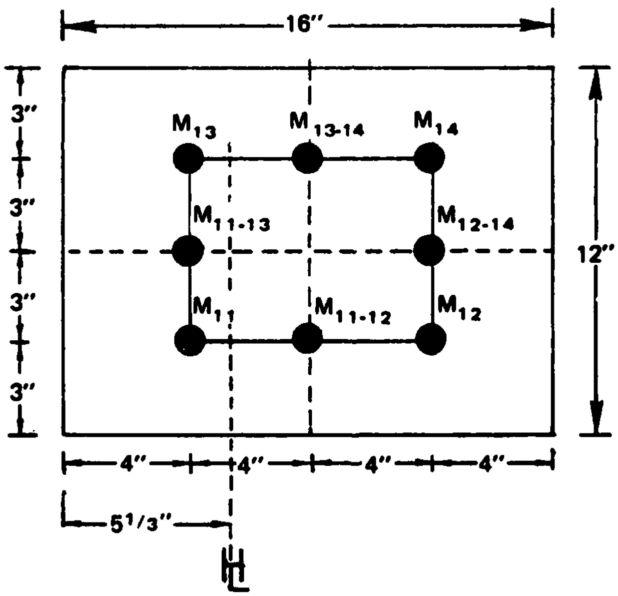

Fig. 4 - Idealization of Flipper Mass Distribution

The Mass Matrix

The missile fuselage is assumed to weigh 1000.0 lbs. and each flipper weighs 5.0 lbs. The ten equal fuselage weights on the half-fuselage are 50.0 lbs. apiece. The flipper idealization is shown in Fig. 4. The flipper center of gravity is assumed to be 0.50 in. forward of the hinge line and 3.86 in. outboard of the fuselage side; the hinge line is located 5.333 in. aft of the leading edge. The pitching moment of inertia about the hinge line is 50.0 lb-in^2 and the rolling moment of inertia about the fuselage side is 137 lb-in^2. The flipper is balanced so it has no product of inertia with respect to the hinge line and fuselage side.

The input to Program MASS consists of the inertial data and the coordinates of the eight mass points. The annotated input deck and program output are shown below in Tables 1 and 2, respectively.

The coupled mass matrix for the four flipper degrees of freedom is:

$$[M] = \begin{bmatrix} M_{11}+(\frac{1}{4})(M_{11-12}+M_{11-13}) & (\frac{1}{4})M_{11-12} & (\frac{1}{4})M_{11-13} & 0 \\ (\frac{1}{4})M_{11-12} & M_{12}+(\frac{1}{4})(M_{11-12}+M_{12-14}) & 0 & (\frac{1}{4})M_{12-14} \\ (\frac{1}{4})M_{11-13} & 0 & M_{13}+(\frac{1}{4})(M_{11-13}+M_{13-14}) & (\frac{1}{4})M_{13-14} \\ 0 & (\frac{1}{4})M_{12-14} & (\frac{1}{4})M_{13-14} & M_{14}+(\frac{1}{4})(M_{12-14}+M_{13-14}) \end{bmatrix}$$

where, from the program output:

$$\begin{aligned} M_{11} &= 4.828472 \\ M_{11-12} &= 0.004167 \\ M_{12} &= 1.695139 \\ M_{11-13} &= -1.584722 \\ M_{13} &= 1.825694 \\ M_{12-14} &= -2.904167 \\ M_{14} &= 2.320139 \\ M_{13-14} &= -1.184722 \end{aligned}$$

Table 1 - Input Cards for Program MASS

```
C      TITLE AND SUBTITLE CARDS
 LEAST SQUARES MASS MATRIX
 MASS ELEMENTS FOR TYPICAL AIR-TO-AIR MISSILE
C      NUMBER OF STRIPS
   1
C      NUMBER OF MASSES
   8
C      INERTIAL PROPERTIES
 5.0          2.5         19.3          50.0          137.0          0.0
C      MASS POINT COORDINATES
 1.333333333 3.0        -2.666666667 3.0        -6.666666667 3.0
 1.333333333 6.0         1.333333333 9.0        -6.666666667 6.0
-6.666666667 9.0        -2.666666667 9.0
```

Table 2 - Output from Program MASS

```
LEAST SQUARES MASS MATRIX
MASS ELEMENTS FOR TYPICAL AIR-TO-AIR MISSILE
                                                MASS MATRIX INPUT DATA
                                                       1 STRIPS

                                              NUMBER OF MASSES BY STRIP

                     1   2   3   4   5   6   7   8   9  10  11  12  13  14  15  16  17  18  19  20

                     8

                                                      STRIP  1

                                            TOTAL MASS M =    .50000000E+01
                                     X UNBALANCE MX BAR =    .25000000E+01
                                     Y UNBALANCE MY BAR =    .19300000E+02
                               Y MOMENT OF INERTIA I SUB Y =  .50000000E+02
                               X MOMENT OF INERTIA I SUB X =  .13700000E+03
                             PRODUCT OF INERTIA I SUB XY =  0.

                              X( 1) =   .13333333E+01      Y( 1) =   .30000000E+01
                              X( 2) =  -.26666667E+01      Y( 2) =   .30000000E+01
                              X( 3) =  -.66666667E+01      Y( 3) =   .30000000E+01
                              X( 4) =   .13333333E+01      Y( 4) =   .60000000E+01
                              X( 5) =   .13333333E+01      Y( 5) =   .90000000E+01
                              X( 6) =  -.66666667E+01      Y( 6) =   .60000000E+01
                              X( 7) =  -.66666667E+01      Y( 7) =   .90000000E+01
                              X( 8) =  -.26666667E+01      Y( 8) =   .90000000E+01

                                          N =  6      NR =  2

        (I)                   BASIC  MASS  AND  REDUNDANT  MASS  GEOMETRY  MATRICES

   .500000E+01         .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01   .100000E+01
   .250000E+01         .133333E+01  -.266667E+01  -.666667E+01   .133333E+01   .133333E+01  -.666667E+01
   .193000E+02         .300000E+01   .300000E+01   .300000E+01   .600000E+01   .900000E+01   .600000E+01
   .500000E+02         .177778E+01   .711111E+01   .444444E+02   .177778E+01   .177778E+01   .444444E+02
   .137000E+03         .900000E+01   .900000E+01   .900000E+01   .360000E+02   .810000E+02   .360000E+02
  0.                   .400000E+01  -.800000E+01  -.200000E+02   .800000E+01   .120000E+02  -.400000E+02

                       .100000E+01   .100000E+01
                      -.666667E+01  -.266667E+01
                       .900000E+01   .900000E+01
                       .444444E+02   .711111E+01
                       .810000E+02   .810000E+02
                      -.600000E+02  -.240000E+02

                                                     MASS MATRIX

         STRIP  1          STRIP

            .48284722E+01
            .41666663E-02
            .18951389E+01
           -.15847222E+01
            .18250944E+01
           -.29041667E+01

            .23201389E+01
           -.11047222E+01
```

As discussed in Appendix B of Ref. 12, the negative values have no particular significance. The numerical value of the coupled mass matrix becomes

$$[M] = \begin{bmatrix} 4.433333 & 0.0010417 & -0.396181 & 0.0 \\ & 0.970139 & 0.0 & -0.726042 \\ & & 1.133333 & -0.296181 \\ & \text{symmetrical} & & 1.297917 \end{bmatrix}$$

Modal Matrices and Frequencies

The modal matrices and frequencies for the fuselage are the same as those used in the MPASES example problems with the addition of the deflections of the four flipper mass points. These are found from numerical interpolation and differentiation among the last four fuselage control point deflections, i.e., mass points 7, 8, 9, and 10, to find the deflection and slope at the flipper hinge line from which the deflections on the flipper follow. The Lagrangian interpolation formula is

$$
\begin{aligned}
h(x) &= h_7 \frac{(x-x_8)(x-x_9)(x-x_{10})}{(x_7-x_8)(x_7-x_9)(x_7-x_{10})} \\
&+ h_8 \frac{(x-x_7)(x-x_9)(x-x_{10})}{(x_8-x_7)(x_8-x_9)(x_8-x_{10})} \\
&+ h_9 \frac{(x-x_7)(x-x_8)(x-x_{10})}{(x_9-x_7)(x_9-x_8)(x_9-x_{10})} \\
&+ h_{10} \frac{(x-x_7)(x-x_8)(x-x_9)}{(x_{10}-x_7)(x_{10}-x_8)(x_{10}-x_9)}
\end{aligned}
$$

The hinge line is at $x = 139.333$ and the remaining coordinates are $x_7 = 97.5$, $x_8 = 112.5$, $x_9 = 127.5$, and $x_{10} = 142.5$, so the interpolation formula becomes

$$h_{HL} = 0.049654\, h_7 - 0.232235\, h_8 + 0.526618\, h_9 + 0.655963\, h_{10}$$

By differentiating the interpolation formula, we obtain the fuselage slope at the hinge line, and the result is

$$h'_{HL} = -0.009633745\, h_7 + 0.048160494\, h_8$$
$$-0.134086420\, h_9 + 0.095559671\, h_{10}$$

From the hinge line deflection and slope the flipper deflections are found from

$$h_{11} = h_{13} = h_{HL} - 1.333333\, h'_{HL}$$

$$h_{12} = h_{14} = h_{HL} + 6.666667\, h'_{HL}$$

This generates the four flipper deflections required to complete the description of the three fuselage modes.

The fourth and fifth vibration modes are bending and torsion of the flipper uncoupled with the fuselage. In bending, if $h_{13} = h_{14} = 1.0$, then $h_{11} = h_{12} = 0.333333$ since the flipper is assumed rigid, and in torsion, if $h_{12} = h_{14} = 1.0$, then $h_{11} = h_{13} = -0.2$, as can be seen from the geometry in Fig. 4.

The control surface deflection mode is similar to the flipper torsion mode except that it corresponds to a unit control surface rotation (one radian, positive trailing edge up), so that $h_{11} = h_{13} = 1.333333$ and $h_{12} = h_{14} = -6.666667$.

The previously assumed frequencies for the fuselage were 45.0, 125.4, and 248.2 Hz. The flipper frequencies are assumed to be 100.0 Hz in bending and 115.0 Hz in torsion.

The damping coefficients in the three fuselage modes are assumed to be $g = 0.01$, 0.02, and 0.03, respectively, and $g = 0.03$ is also assumed for both flipper modes.

The rigid body modes require the addition of the four flipper mass points: four unit plunging displacements, and four x-coordinates for pitching about the fuselage nose, $x_{11} = x_{13} = 138.0$ and $x_{12} = x_{14} = 146.0$.

Servo System

The servo system block diagram is shown in Fig. 5. The subscripts on the outputs are the same as the servo element numbers. The system consists of three loops with rate, attitude, and acceleration feedbacks. The transfer functions and their corresponding differential equations and numerical values are discussed in the following sections for each component in the three loops.

Control Surface Actuator - The actuator servo is assumed to move the control surface shaft with an angular rate proportional to the difference between the desired and actual angular displacements. Its transfer function is

$$Y_{FS} = \frac{\delta}{e_5} = \frac{1}{s/K_v + 1}$$

and the corresponding differential equation is

$$(1/K_v)\dot{\delta} + \delta - e_5 = 0$$

The numerical value $1/K_v = 0.016$ sec. is used in the analysis.

Servo Element No. 1 - The rate gyro transfer function is

$$e_1/\dot{\theta} = \frac{K_g}{s^2/\omega_g^2 + (2\zeta_g/\omega_g)s + 1}$$

and the differential equation is

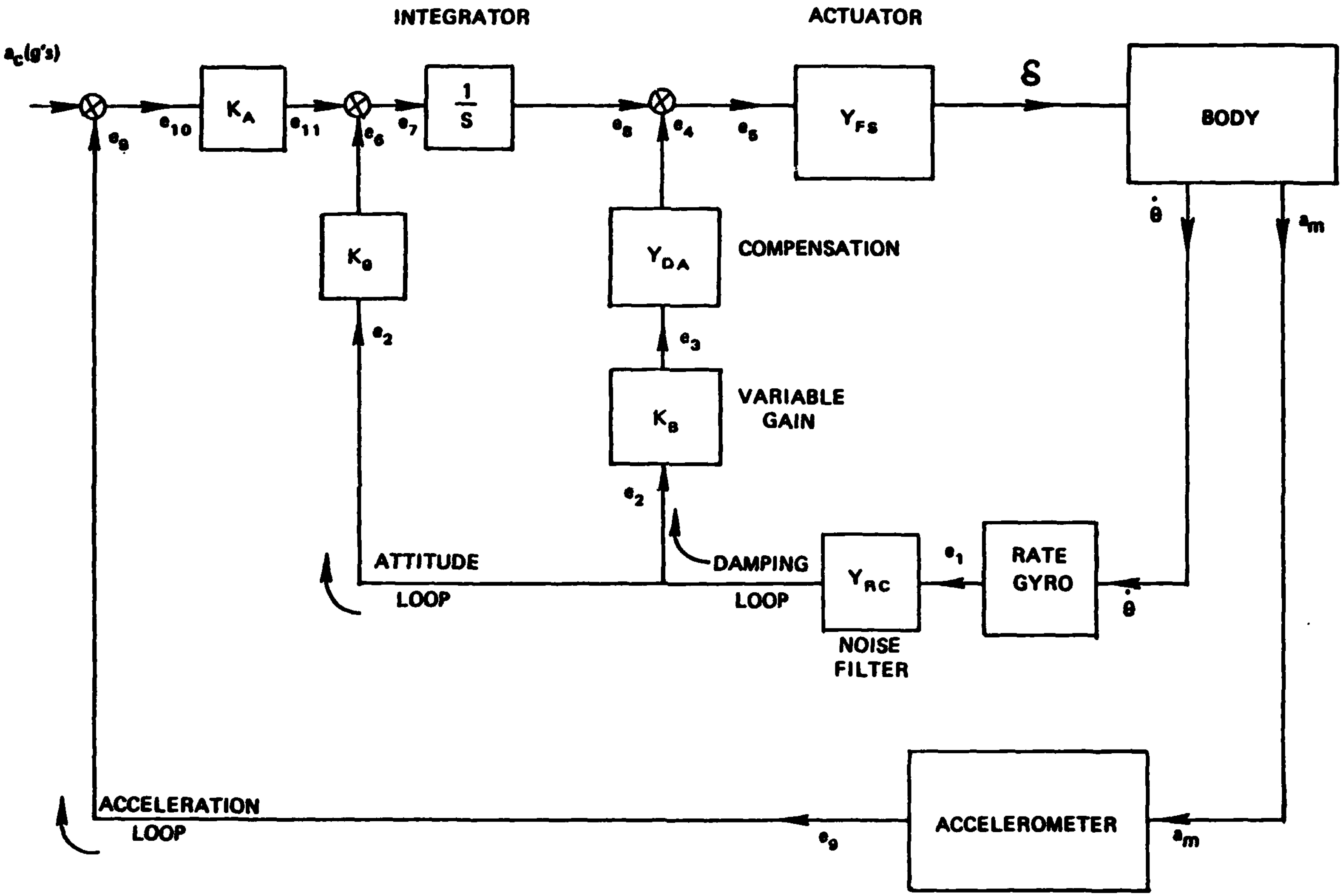

Fig. 5 - Servo System Block Diagram

$$(1/\omega_g^2)\ddot{e}_1 + (2\zeta_g/\omega_g)\dot{e}_1 + e_1 - K_g\dot{\theta} = 0$$

The numerical values are taken as $1/\omega_g^2 = 0.659 \times 10^{-5}$ sec^2, $2\zeta_g/\omega_g = 0.359 \times 10^{-2}$ sec, and $K_g = 1.0$ deg/sec per deg/sec.

The numerical differentiation matrix to obtain the rate $\dot{\theta}$ is based on a cubic spline fit among the deflections of mass points 2, 3, 4, and 5. The differentiation matrix is (see Ref. 11)

$$\lfloor D \rfloor = (1/24\ell)\lfloor 0 \quad +1 \quad -27 \quad +27 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \rfloor$$

where $\ell = 15.0$ in., the distance between fuselage mass points. The coefficient $1/24\ell = 0.002777778$ is input as the rate gyro gain, and the integer elements of the differentiation matrix are input directly.

Servo Element No. 2 - The noise filter transfer function and differential equation are

$$Y_{RC} = e_2/e_1 = \frac{1}{\tau s + 1}$$

and

$$\tau\dot{e}_2 + e_2 - e_1 = 0$$

where $\tau = 0.001$ sec.

Servo Element No. 3 - For the variable gain amplifier we use

$$e_3/e_2 = K_B$$

or

$$e_3 - K_B e_2 = 0$$

where $K_B = 0.1$ deg/deg/sec for a Mach number of 3.0 at sea level. Although the aero-servo-elastic stability analysis covers a range of supersonic Mach numbers, a constant value of K_B is used.

Servo Element No. 4 - The compensating damping amplifier equations are

$$Y_{DA} = e_4/e_3 = \frac{T_d s + 1}{s^2/\omega_d^2 + (2\zeta_d/\omega_d)s + 1}$$

and

$$(1/\omega_d^2)\ddot{e}_4 + (2\zeta_d/\omega_d)\dot{e}_4 + e_4 - T_d\dot{e}_3 - e_3 = 0$$

where $1/\omega_d^2 = 0.253 \times 10^{-5}$ sec^2, $2\zeta_d/\omega_d = 0.223 \times 10^{-2}$ sec, and $T_d = 0.016$ sec.

Servo Element No. 5 - The equation for the rate loop feedback junction is

$$e_5 = e_8 - e_4$$

or

$$e_5 + e_4 - e_8 = 0$$

Servo Element No. 6 - The attitude amplifier gain is

$$e_6/e_2 = K_\theta$$

or

$$e_6 - K_\theta e_2 = 0$$

where $K_{\theta} = 2.0$ deg/sec per deg/sec.

Servo Element No. 7 - The equation for the attitude loop feedback junction is

$$e_7 = e_{11} - e_6$$

or

$$e_7 + e_6 - e_{11} = 0$$

Servo Element No. 8 - The electronic integrator in the attitude loop has the equation

$$e_8/e_7 = 1/s$$

or

$$\dot{e}_8 - e_7 = 0$$

Servo Element No. 9 - The accelerometer has the transfer function

$$e_9/a_m = \frac{1}{s^2/\omega_a^2 + (2\zeta_a/\omega_a)s + 1}$$

and equation of motion

$$(1/\omega_a^2)\ddot{e}_9 + (2\zeta/\omega_a)\dot{e}_9 + e_9 - a_m = 0$$

where $1/\omega_a^2 = 1.013 \times 10^{-5}$ sec^2 and $2\zeta_a/\omega_a = 0.6366 \times 10^{-2}$ sec.

The numerical interpolation matrix to obtain the acceleration a_m is also based on a cubic spline fit among the deflections of mass points 2, 3, 4, and 5. The interpolation matrix is (see Ref. 11)

$$\lfloor H \rfloor = (1/16)\lfloor 0 \quad -1 \quad +9 \quad +9 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \rfloor$$

The coefficient $1/16 = 0.0625$ is input as the accelerometer gain, and the integer elements of the interpolation matrix are input directly.

Servo Element No. 10 - The acceleration loop feedback equation is

$$e_{10} = a_c - e_9$$

which reduces to

$$e_{10} + e_9 = 0$$

for the stability problem.

Servo Element No. 11 - The acceleration loop gain is

$$e_{11}/e_{10} = K_a$$

or

$$e_{11} - K_a e_{10} = 0$$

where $K_a = 4.0$ deg/sec per g = 0.002170 sec/ft.

Servoelastic Stability Analysis

The input data cards are prepared according to the MPASES input instructions in Sect. III. The data are found in the previous sections of this appendix with the exception of the fuselage vibration modes which are found in the original MPASES example problem; the vibration modes for the present example are found by combining the previous fuselage modes with the additional flipper mass point deflections derived above in this report. The shift eigenvalue $\gamma_0 = +100.0$ is chosen to scale [A] to be of the same order of magnitude as [B] and with the same sign. The input data deck, with comments added, is reproduced in Table 3.

The printed output is reproduced in Table 4 except that only the modes and deflections corresponding to eigenvalues of practical interest are shown. (The modes and deflections corresponding to zero or infinite eigenvalues are not shown.)

The stability analysis results come from the solution of a 30th order eigenvalue problem. The order of the eigenvalue problem is found from

$$N = 2(m+n+c) + ns + s$$

where N = order of the eigenvalue problem

m = number of flexible modes

n = number of rigid body modes

c = number of control surfaces

ns = number of servo elements

s = number of second order servo elements.

In this example, m = 5, n = 2, c = 1, ns = 11, and s = 3, so that N = 30. The stability analysis results are printed for 22 modes since 8 of the eigenvalues are complex conjugate pairs. Only modes 4-7, and 16-22 are of practical interest. The seven oscillatory and four non-oscillatory solutions are all seen to be stable. The motions involved in the modes are summarized as follows.

Mode No.	Dominant Motion
4	Actuator
5 & 6	Accelerometer (critically damped)
7	Noise filter
16	Rate gyro
17	First body bending
18	Compensating damping amplifier
19	Flipper bending
20	Flipper torsion
21	Second body bending
22	Third body bending

Table 3 - Input Cards for Program MPASES for Servoelastic Stability Analysis

```
C      TITLE CARDS
 TYPICAL AIR-TO-AIR MISSILE
 SERVOELASTIC ANALYSIS
C      CONTROL CARD
    2   14    5    2    1   11    0    0    0    0   50    1    0    1
C      SHIFT EIGENVALUE
 100.0
C      MASS MATRIX
C      1ST ROW
 50.0        0.0         0.0         0.0         0.0         0.0
 0.0         0.0         0.0         0.0         0.0         0.0
 0.0         0.0
C      2ND ROW
 50.0        0.00        0.0         0.0         0.0         0.0
 0.0         0.0         0.0         0.0         0.0         0.0
 0.0
C      3RD ROW
 50.0        0.0         0.00        0.0         0.0         0.0
 0.0         0.0         0.0         0.0         0.0         0.0
C      4TH ROW
 50.0        0.0         0.0         0.0         0.0         0.0
 0.0         0.0         0.0         0.0         0.0
C      5TH ROW
 50.0        0.0         0.0         0.0         0.0         0.0
 0.0         0.0         0.0         0.0
C      6TH ROW
 50.0        0.0         0.0         0.0         0.0         0.0
 0.0         0.0         0.0
C      7TH ROW
 50.0        0.0         0.0         0.0         0.0         0.0
 0.0         0.0
C      8TH ROW
 50.0        0.0         0.0         0.0         0.0         0.0
 0.0
C      9TH ROW
 50.0        0.0         0.0         0.0         0.0         0.0
C      10TH ROW
 50.0        0.0         0.0         0.0         0.0
C      11TH ROW
 4.4333333   0.0010417  -0.3961806   0.0
C      12TH ROW
 0.9701389   0.0        -0.7260417
C      13TH ROW
 1.1333332  -0.2961806
C      14TH ROW
 1.2979166
C      CONTROL SURFACE DEFLECTION MODE
 0.0         0.0         0.0         0.0         0.0         0.0
 0.0         0.0         0.0         0.0         1.333333333-6.666666667
 1.333333333-6.666666667
C        DAMPING COEFFICIENTS, G
 0.01        0.02        0.03        0.03        0.03
C      MODAL VIBRATION FREQUENCIES
 45.0        125.4       248.2       100.0       115.0
C      VIBRATION MODE SHAPES
C      1ST MODE
 1.0         0.411195   -0.122424   -0.532567   -0.756204   -0.756204
-0.532467   -0.122424    0.411195    1.0         0.821620    1.138881
 0.821620    1.138881
C      2ND MODE
 1.0        -0.150871   -0.910842   -0.990216   -0.419043    0.419043
```

```
 0.990215     0.910842     0.150871     -1.0          -0.630256    -1.281954
-0.630256    -1.281954
C      3RD MODE
-0.803195     0.645103     1.0           0.105986     -0.947894    -0.947893
 0.105986     1.0          0.645103     -0.803195     -0.259298    -1.188204
-0.259298    -1.188204
C      4TH MODE
 0.0          0.0          0.0           0.0           0.0          0.0
 0.0          0.0          0.0           0.0          .3333333333 .3333333333
 1.0          1.0
C      5TH MODE
 0.0          0.0          0.0           0.0           0.0          0.0
 0.0          0.0          0.0           0.0          -0.2          1.0
-0.2          1.0
C      RIGID BODY MODES
C      PLUNGE MODE
 1.0          1.0          1.0           1.0           1.0          1.0
 1.0          1.0          1.0           1.0           1.0          1.0
 1.0          1.0
C      PITCH MODE
 7.5          22.5         37.5          52.5          67.5         82.5
 97.5         112.5        127.5         142.5         138.0        146.0
 138.0        146.0
C      CONTROL SURFACE OUTPUT
 1.0          0.016        0.0
C      NUMBER OF INPUTS TO CONTROL SURFACE
    1
C      CONTROL SURFACE INPUTS FROM SERVOS
    5    0   -1.0
C      OUTPUT FROM SERVO ELEMENT
    1    2    1.0          0.359     -2 0.659     -5
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      INPUT FROM BODY PITCH RATE
    0   -1   -2.7777778-3
C      DIFFERENTIATION MATRIX FOR RATE GYRO
 0.0          1.0         -27.0          27.0          -1.0          0.0
 0.0          0.0          0.0           0.0           0.0          0.0
 0.0          0.0
C      OUTPUT FROM SERVO ELEMENT
    2    1    1.0          0.001
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      INPUTS TO SERVO ELEMENT
    1    0   -1.0
C      OUTPUT FROM SERVO ELEMENT
    3    0    1.0
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      INPUTS TO SERVO ELEMENT
    2    0   -0.1
C      OUTPUT FROM SERVO ELEMENT
    4    2    1.0          0.223     -2 0.253     -5
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      INPUTS TO SERVO ELEMENT
    3    1   -1.0         -0.016
C      OUTPUT FROM SERVO ELEMENT
    5    0    1.0
```

```
C      NUMBER OF INPUTS TO SERVO ELEMENT
    2
C      INPUTS TO SERVO ELEMENT
    4    0    1.0
    8    0   -1.0
C      OUTPUT FROM SERVO ELEMENT
    6    0    1.0
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      INPUTS TO SERVO ELEMENT
    2    0   -2.0
C      OUTPUT FROM SERVO ELEMENT
    7    0    1.0
C      NUMBER OF INPUTS TO SERVO ELEMENT
    2
C      INPUTS TO SERVO ELEMENT
    6    0    1.0
   11    0   -1.0
C      OUTPUT FROM SERVO ELEMENT
    8    1    0.0            1.0
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      INPUTS TO SERVO ELEMENT
    7    0   -1.0
C      OUTPUT FROM SERVO ELEMENT
    9    2    1.0           0.6366       -2 1.013     -5
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      INPUT FROM BODY ACCELERATION
    0   -2   -0.0625
C      INTERPOLATION MATRIX FOR ACCELEROMETER
 0.0           -1.0            9.0            9.0           -1.0            0.0
 0.0            0.0            0.0            0.0            0.0            0.0
 0.0            0.0
C      OUTPUT FROM SERVO ELEMENT
   10    0    1.0
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      INPUTS TO SERVO ELEMENT
    9    0    1.0
C      OUTPUT FROM SERVO ELEMENT
   11    0    1.0
C      NUMBER OF INPUTS TO SERVO ELEMENT
    1
C      OUTPUT FROM SERVO ELEMENT
   10    0   -0.002170
```

Table 4 - Output from Program MPASES for Servoelastic Stability Analysis

```
TYPICAL AIR-TO-AIR MISSILE

SERVOELASTIC ANALYSIS

                              SERVO ELASTIC STABILITY ANALYSIS

                                 14 DEGREES OF FREEDOM
                                  5 FLEXIBLE MODES
                                  2 RIGID BODY MODES
                                  1 CONTROL SURFACES
                                 11 SERVO ELEMENTS

                             SHIFT EIGENVALUE(GAMMA) =  1.000E+02

                                 50 MODES REQUESTED

   UPPER TRIANGLE OF WEIGHT MATRIX

   ROW  1
        5.00000E+01   0.        0.        0.        0.        0.        0.        0.
        0.            0.        0.        0.        0.        0.

   ROW  2
        5.00000E+01   0.        0.        0.        0.        0.        0.        0.
        0.            0.        0.        0.        0.

   ROW  3
        5.00000E+01   0.        0.        0.        0.        0.        0.        0.
        0.            0.        0.        0.

   ROW  4
        5.00000E+01   0.        0.        0.        0.        0.        0.        0.
        0.            0.        0.

   ROW  5
        5.00000E+01   0.        0.        0.        0.        0.        0.        0.
        0.            0.

   ROW  6
        5.00000E+01   0.        0.        0.        0.        0.        0.        0.
        0.
```

```
ROW  7
   5.00000E+01   0.            0.            0.            0.            0.            0.            0.

ROW  8
   5.00000E+01   0.            0.            0.            0.            0.            0.

ROW  9
   5.00000E+01   0.            0.            0.            0.            0.

ROW 10
   5.00000E+01   0.            0.            0.            0.

ROW 11
   6.45333E+00   1.04170E-03  -3.96101E-01   0.

ROW 12
   9.70119E-01   0.           -7.26042E-01

ROW 13
   1.13333E+00  -2.96101E-01

ROW 14
   1.29792E+00


RIGID BODY CONTROL SURFACE MODES


MODE 1

   0.            0.            0.            0.            0.            0.            0.            0.
   0.            0.            1.33333E+00  -6.66667E+00   1.33333E+00  -6.66667E+00


FLEXIBLE MODES


MODE 1 - FREQUENCY =      45.000 CPS
         STRUCTURAL DAMPING COEFFICIENT =  .010

   1.00000E+00   8.11195E-01  -1.22024E-01  -5.32567E-01  -7.56204E-01  -7.56204E-01  -5.32467E-01  -1.22024E-01
   8.11195E-01   1.00000E+00   8.21620E-01   1.13888E+00   8.21620E-01   1.13888E+00

MODE 2 - FREQUENCY =     125.400 CPS
         STRUCTURAL DAMPING COEFFICIENT =  .020

   1.00000E+00  -1.50871E-01  -9.10842E-01  -9.90216E-01  -4.19043E-01   4.19043E-01   9.90215E-01   9.10842E-01
   1.50871E-01  -1.00000E+00  -6.30256E-01  -1.28195E+00  -6.30256E-01  -1.28195E+00

MODE 3 - FREQUENCY =     248.200 CPS
         STRUCTURAL DAMPING COEFFICIENT =  .030

  -8.03195E-01   6.45103E-01   1.00000E+00   1.05986E-01  -9.47894E-01  -9.47893E-01   1.05986E-01   1.00000E+00
   6.45103E-01  -8.03195E-01  -2.59290E-01  -1.18820E+00  -2.59290E-01  -1.18820E+00
```

```
MODE 4 - FREQUENCY =     100.000 CPS
         STRUCTURAL DAMPING COEFFICIENT =  .030

   0.           0.           0.             0.             0.             0.             0.           0.
   0.           0.           3.33333E-01    3.33333E-01    1.00000E+00    1.00000E+00
MODE 5 - FREQUENCY =     115.000 CPS
         STRUCTURAL DAMPING COEFFICIENT =  .030

   0.           0.           0.             0.             0.             0.             0.           0.
   0.           0.          -2.00000E-01    1.00000E+00   -2.00000E-01    1.00000E+00


RIGID BODY MODES


MODE 1

   1.00000E+00  1.00000E+00  1.00000E+00  1.00000E+00  1.00000E+00  1.00000E+00  1.00000E+00  1.00000E+00
   1.00000E+00  1.00000E+00  1.00000E+00  1.00000E+00  1.00000E+00  1.00000E+00
MODE 2

   7.50000E+00  2.25000E+01  3.75000E+01  5.25000E+01  6.75000E+01  8.25000E+01  9.75000E+01  1.12500E+02
   1.27500E+02  1.42500E+02  1.38000E+02  1.46000E+02  1.38000E+02  1.46000E+02
```

CONTROL SYSTEM DESCRIPTION

DIFFERENTIAL EQUATION FOR	VARIABLE	COEFFICIENTS 2ND ORDER	1ST ORDER	0 ORDER	GYRO/ACCEL. GAIN FACTOR
CONTROL SURF. 1	CONTROL SURF. 1	0.	1.6000E-02	1.0000E+00	
	SERVO ELEMENT 5	0.	0.	-1.0000E+00	
SERVO ELEMENT 1 (RATE GYRO)	SERVO ELEMENT 1	6.5900E-06	3.5900E-03	1.0000E+00	
	BODY ANGULAR RATE				-2.7778E-03
SERVO ELEMENT 2	SERVO ELEMENT 2	0.	1.0000E-03	1.0000E+00	
	SERVO ELEMENT 1	0.	0.	-1.0000E+00	
SERVO ELEMENT 3	SERVO ELEMENT 3	0.	0.	1.0000E+00	
	SERVO ELEMENT 2	0.	0.	-1.0000E-01	
SERVO ELEMENT 4	SERVO ELEMENT 4	2.5300E-06	2.2300E-03	1.0000E+00	
	SERVO ELEMENT 3	0.	-1.6000E-02	-1.0000E+00	
SERVO ELEMENT 5	SERVO ELEMENT 5	0.	0.	1.0000E+00	
	SERVO ELEMENT 4	0.	0.	1.0000E+00	
	SERVO ELEMENT 8	0.	0.	-1.0000E+00	
SERVO ELEMENT 6	SERVO ELEMENT 6	0.	0.	1.0000E+00	
	SERVO ELEMENT 2	0.	0.	-2.0000E+00	
SERVO ELEMENT 7	SERVO ELEMENT 7	0.	0.	1.0000E+00	
	SERVO ELEMENT 6	0.	0.	1.0000E+00	
	SERVO ELEMENT 11	0.	0.	-1.0000E+00	
SERVO ELEMENT 8	SERVO ELEMENT 8	0.	1.0000E+00	0.	
	SERVO ELEMENT 7	0.	0.	-1.0000E+00	
SERVO ELEMENT 9 (ACCELEROMETER)	SERVO ELEMENT 9	1.0130E-05	6.3600E-03	1.0000E+00	
	BODY ACCELERATION				-6.2500E-02
SERVO ELEMENT 10	SERVO ELEMENT 10	0.	0.	1.0000E+00	
	SERVO ELEMENT 9	0.	0.	1.0000E+00	
SERVO ELEMENT 11	SERVO ELEMENT 11	0.	0.	1.0000E+00	
	SERVO ELEMENT 10	0.	0.	-1.0002E-04	

GYRO/ACCEL.   INPUT DIFFERENTIATION/INTERPOLATION ROW VECTOR

```
SERVO ELEMENT  1    0.     1.0000E+00  -2.7000E+01   2.7000E+01  -1.0000E+00   0.     0.     0.
                    0.     0.           0.           0.           0.           0.
SERVO ELEMENT  9    0.    -1.0000E+00   9.0000E+00   9.0000E+00  -1.0000E+00   0.     0.     0.
                    0.     0.           0.           0.           0.           0.
```

SERVO ELEMENT	MAXIMUM ORDER	ROW ASSIGNMENT IN A AND/OR B	COLUMN ASSIGNMENT IN A AND/OR B	EIGENVECTOR ELEMENT VELOCITY	EIGENVECTOR ELEMENT DISPLACEMENT
1	2	9	9 AND 20	9	20
2	1	10	23		23
3	1	11	24		24
4	2	12	10 AND 21	10	21
5	0	13	26		26
6	0	14	27		27
7	0	15	28		28
8	1	16	25		25
9	2	17	11 AND 22	11	22
10	0	18	29		29
11	0	19	30		30

A MATRIX

6.4548312E+00	-9.7467163E-02	-5.4251265E-02	5.8286490E-02	-9.4810105E-04	1.3297506E-01	1.0515037E+01	6.3206789E-03
0.	0.	0.	1.0250605E+01	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
-9.7467163E-02	9.4365765E+00	5.2963002E-02	-4.9246918E-02	-1.0377622E-02	-1.0849617E-01	-1.5186298E+01	6.9184149E-02
0.	0.	0.	0.	1.4870373E+02	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
-5.4251265E-02	5.2963002E-02	9.2743418E+00	-2.7601411E-02	-2.2240190E-02	-5.5331789E-02	-7.8579189E+00	1.4826793E-01
0.	0.	0.	0.	0.	4.3389635E+02	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
5.8286490E-02	-4.9246918E-02	-2.7601411E-02	5.2549083E-02	-2.9003099E-09	6.6651460E-02	9.2867700E+00	1.9313102E-08
0.	0.	0.	0.	0.	0.	9.9090384E-01	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
-9.4810105E-04	-1.0377622E-02	-2.2240190E-02	-2.9003099E-09	3.4966118E-02	-1.1655376E-02	-1.3908750E+00	-2.3310745E-01
0.	0.	0.	0.	0.	0.	0.	7.5796816E-01
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
1.3297506E-01	-1.0849617E-01	-5.5331789E-02	6.6651460E-02	-1.1655376E-02	1.5695984E+01	1.1871128E+03	7.7702509E-02
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
1.0515037E+01	-1.5186298E+01	-7.8579189E+00	9.2867700E+00	-1.3908750E+00	1.1871128E+03	1.1925925E+05	9.2724999E+00
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	1.6000000E-02	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
6.5900000E-06	0.	0.	2.7517950E-02	5.2801278E-03	6.2626059E-02	0.	0.
-4.3021142E-16	-1.0000000E+00	0.	3.5900000E-03	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	1.0000000E-03	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	2.5300000E-06	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	2.2300000E-03	0.	0.	-1.6000000E-02
0.	0.	0.	0.	0.	0.		

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
1.0000000E+00	0.	0.	0.	0.	0.		
3.4606938E-01	1.0337255E+00	-6.4104156E-01	0.	0.	-1.0000000E+00	-4.5000000E+01	0.
0.	0.	1.0130000E-05	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	6.3600000E-03	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.0000000E+00	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.0000000E+00	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.0000000E+00	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	1.0000000E+00	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.0000000E+00
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
1.0000000E+00	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	1.0000000E+00	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	1.0000000E+00	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.0000000E+00	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.0000000E+00	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.0000000E+00	0.	0.
0.	0.	0.	0.	0.	0.		

B MATRIX

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	5.1602370E+05	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	5.0582685E+06	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	2.2555185E+07	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	2.0753442E+04	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	1.0255083E+04
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	1.0000000E+00	0.	0.	0.	0.	0.
0.	-1.0000000E+00	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	1.0000000E+00	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-1.0000000E+00	0.	0.	1.0000000E+00	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-1.0000000E-01	1.0000000E+00
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.0000000E+00	0.	0.	-1.0000000E+00
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.0000000E+00	0.	0.	0.
-1.0000000E+00	1.0000000E+00	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	-2.0000000E+00	0.
0.	0.	1.0000000E+00	0.	0.	0.		

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	1.0000000E+00	1.0000000E+00	0.	-1.0000000E+00		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	-1.0000000E+00	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.0000000E+00	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	1.0000000E+00	0.	0.
0.	0.	0.	0.	1.0000000E+00	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.0002120E-04	1.0000000E+00		
-1.0000000E+00	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		

0.	-1.0000000E+00	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	-1.0000000E+00	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	-1.0000000E+00	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	-1.0000000E+00	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	-1.0000000E+00	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	-1.0000000E+00	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	-1.0000000E+00
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
-1.0000000E+00	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	-1.0000000E+00	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	-1.0000000E+00	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.		

STABILITY ANALYSIS RESULTS

MODE	EIGENVALUE-R MU-RPS	EIGENVALUE-I OMEGA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-2.501110E-11	0.	0.	3.980641E-12		2.771358E+10
2	-3.001333E-11	0.	0.	4.776769E-12		2.309465E+10
3	-1.320640E-06	0.	0.	2.101863E-07		5.248572E+05
4	-6.249950E+01	0.	0.	9.947104E+00		1.109044E-02
5	-3.093045E+02	0.	0.	4.924006E+01		2.240407E-03
6	-3.189769E+02	0.	0.	5.076675E+01		2.173032E-03
7	-9.985427E+02	0.	0.	1.589230E+02		6.941588E-04
8	3.565272E+15	0.	0.	5.674305E+14		-1.944164E-16
9	-9.007199E+15	0.	0.	1.433540E+15		7.695480E-17
10	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
11	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
12	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
13	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
14	-1.000000E+38	0.	0.	1.591549E+37		6.931472E-39
15	6.602160E-07	2.957091E-06	4.706357E-07	4.822231E-07	-2.179007E-01	-1.049878E+06
16	-2.699049E+02	2.811908E+02	4.475291E+01	6.203306E+01	6.924807E-01	2.568116E-03
17	-1.451224E+00	2.824120E+02	4.494736E+01	4.494795E+01	5.138599E-03	4.776293E-01
18	-4.435253E+02	4.440194E+02	7.066788E+01	9.988389E+01	7.067131E-01	1.562813E-03
19	-9.571353E+00	6.348437E+02	1.010385E+02	1.010500E+02	1.507500E-02	7.241893E-02
20	-1.101639E+01	7.240024E+02	1.152413E+02	1.152546E+02	1.521252E-02	6.291960E-02
21	-8.108520E+00	7.931509E+02	1.262339E+02	1.262405E+02	1.022264E-02	8.548381E-02
22	-2.369782E+01	1.562317E+03	2.486505E+02	2.486791E+02	1.516664E-02	2.924941E-02

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTOR FOR MODE 4

4.0549204E-08	0.	5.5798854E-04	0.	3.0744616E-06	0.	-1.8234730E-05	0.
-5.9591332E-03	0.	-4.4874887E-04	0.	1.3738380E-05	0.	-1.2011885E-01	0.
-1.0454439E-03	0.	-1.0289726E-09	0.	1.0000000E+00	0.	-6.4879247E-08	0.
-8.9278884E-08	0.	-4.9191781E-08	0.	2.9175802E-07	0.	9.5346897E-05	0.
7.1799949E-09	0.	-2.1981585E-07	0.	1.9219170E-03	0.	1.8727877E-05	0.
1.6463688E-11	0.	-1.6000129E-02	0.	1.7843059E-05	0.	1.7843060E-09	0.
1.5453561E-08	0.	1.5437697E-08	0.	3.5080119E-05	0.	-9.6543980E-07	0.
1.6000129E-02	0.	3.4720279E-05	0.				

EIGENVECTOR FOR MODE 5

6.2800984E-10	0.	1.4506925E-09	0.	9.0817947E-10	0.	-4.3603665E-09	0.
-1.8383346E-06	0.	-5.6324599E-09	0.	1.7260964E-10	0.	-1.7614219E-06	0.
-5.3625851E-08	0.	5.5542152E-08	0.	1.0000000E+00	0.	-2.0298662E-12	0.
-4.8182529E-12	0.	-2.9354399E-12	0.	1.4093083E-11	0.	5.9419101E-09	0.
1.8205375E-11	0.	-5.5616771E-13	0.	5.6933112E-09	0.	1.7333017E-10	0.
-1.7952471E-10	0.	-3.2322245E-03	0.	2.5098014E-10	0.	2.5098006E-11	0.
-2.2068965E-08	0.	-2.2469440E-08	0.	5.0196029E-10	0.	7.0134252E-06	0.
3.2322245E-03	0.	7.0134272E-06	0.				

EIGENVECTOR FOR MODE 6

5.9858532E-10	0.	1.4529134E-09	0.	8.9855120E-10	0.	-4.2194927E-09	0.
-1.8074411E-06	0.	-5.2090857E-09	0.	1.5913847E-10	0.	-1.6447416E-06	0.
-4.8164091E-08	0.	5.3144429E-08	0.	1.0000000E+00	0.	-1.8763288E-12	0.
-4.5549187E-12	0.	-2.7914985E-12	0.	1.3228207E-11	0.	5.6663696E-09	0.
1.6332861E-11	0.	-4.9890277E-13	0.	5.1503028E-09	0.	1.5099553E-10	0.
-1.0600901E-10	0.	-3.1350230E-03	0.	2.2171867E-10	0.	2.2171855E-11	0.
-2.1326172E-08	0.	-2.1159563E-08	0.	4.4343735E-10	0.	6.8025566E-06	0.
3.1350230E-03	0.	6.8030000E-06	0.				

EIGENVECTOR FOR MODE 7

-6.4120980E-06	0.	-3.5586580E-05	0.	-3.6392119E-05	0.	5.5904401E-05	0.
1.0223284E-01	0.	2.9887284E-05	0.	-9.0092512E-07	0.	2.3040887E-02	0.
-4.3437353E-04	0.	3.4448004E-01	0.	1.0000000E+00	0.	6.4214486E-09	0.
3.5630442E-08	0.	3.0445230E-08	0.	-5.5985998E-08	0.	-1.0238284E-04	0.
-2.9930983E-08	0.	9.8824872E-10	0.	-2.3874434E-05	0.	4.3500747E-07	0.
-3.4490279E-04	0.	-1.0014594E-03	0.	2.9650181E-04	0.	2.9850161E-05	0.
5.9569857E-07	0.	3.4557848E-04	0.	5.9700363E-06	0.	-5.9483046E-06	0.
1.0614594E-03	0.	2.1731670E-06	0.				

EIGENVECTOR FOR MODE 16

-2.2525847E-06	6.7214157E-07	-3.0132701E-06	5.4036712E-06	-1.1093501E-06	3.2729100E-06	1.3567979E-05	-1.5032648E-05
1.7023439E-03	-6.4946487E-03	1.2375321E-05	3.7476692E-06	-3.7800330E-07	-1.1628471E-07	3.5177637E-03	6.6922577E-05
-7.2744676E-03	-1.0172707E-02	1.1903676E-02	-1.5601106E-02	1.0000000E+00	0.	5.2461808E-09	2.9752560E-09
1.5355492E-08	-4.0230765E-09	6.0289361E-09	-3.7614989E-09	-5.1930490E-08	1.5943420E-09	-1.5040130E-05	8.3881845E-06
-1.5649998E-08	-2.9564453E-08	4.5634642E-10	9.0026419E-10	-6.1260034E-06	-6.6301077E-06	-5.9049564E-06	3.1538101E-05
-5.0208474E-05	5.7159396E-06	-1.7706611E-03	-1.8509512E-03	7.4448474E-06	4.0329921E-05	7.4448473E-07	4.0329921E-06
-1.2225809E-07	1.5659315E-07	5.0158215E-05	-5.5593465E-06	1.4869695E-05	8.0659842E-05	-1.1034340E-05	-7.0043278E-05
1.7706611E-03	1.8509512E-03	3.8553546E-05	4.0165641E-06				

EIGENVECTOR FOR MODE 17

7.1002983E-06	6.4709439E-05	2.3657983E-08	-6.2192060E-08	1.5864169E-08	8.7551507E-09	1.8622272E-06	1.7374346E-05
-3.2711595E-05	-4.2678275E-05	2.2660958E-07	1.1302385E-06	-5.9924454E-09	-2.5921340E-08	2.7144974E-05	3.5554156E-05
1.4910910E-04	-4.3690312E-04	1.3732665E-04	-1.4955070E-04	1.0000000E+00	0.	2.2899548E-07	-2.6346610E-08
-2.2004164E-10	-6.2637192E-11	3.0711809E-11	-5.6331551E-11	6.1485641E-08	-6.9899494E-09	-1.5122929E-07	1.1660622E-07
3.9970527E-09	-6.2316222E-10	-9.1673890E-11	2.1091261E-11	1.2539715E-07	-9.6702524E-08	-1.5497107E-06	-5.2061975E-07
-5.3283179E-07	-4.6352858E-07	-1.8195122E-05	-3.5408253E-03	-1.5733961E-06	-7.5762689E-08	-1.5733961E-07	-7.5782883E-09
2.7084946E-08	-1.1424607E-08	5.5971674E-07	4.7210397E-07	-3.1467922E-06	-1.5156578E-07	3.1862756E-06	7.8351567E-06
1.8195122E-05	3.5408253E-03	3.9403416E-08	7.0835909E-06				

EIGENVECTOR FOR MODE 18

-2.1133006E-06	3.0167345E-07	-6.9102225E-06	7.6693614E-06	-2.0893591E-06	6.2790729E-06	2.5909410E-05	-9.4403624E-06
1.9211271E-03	-1.2950362E-02	9.7218534E-06	1.4177627E-06	-2.9556002E-07	-4.5046834E-08	2.8100348E-03	-1.5160002E-03
-2.4541343E-04	1.0143408E-04	6.3475992E-03	-2.9215330E-02	1.0000000E+00	0.	2.7874632E-09	1.9751165E-03
1.6427343E-08	-8.4418073E-10	9.4313086E-09	-4.7153165E-09	-3.9094965E-08	-1.8036532E-06	-1.6702675E-05	1.2417350E-05
-9.3442612E-09	-1.2550252E-08	2.8204113E-10	3.6392070E-10	-4.8750760E-06	-1.4583798E-06	3.9116542E-07	1.6197864E-07
-4.0003216E-05	2.5742647E-05	-1.1260763E-03	-1.1273300E-03	5.7134984E-07	-1.6484842E-07	5.7134984E-08	-1.6484842E-08
1.6046904E-09	-4.5924081E-09	4.0084808E-05	-2.5747439E-05	1.1427497E-06	-3.2969684E-07	1.3007858E-06	2.7760842E-06
1.1260763E-03	1.1273306E-03	2.4435855E-06	2.4463074E-06				

EIGENVECTOR FOR MODE 19

-6.2203919E-06	8.6594742E-06	-5.2130320E-06	6.8646177E-06	-2.9511088E-07	4.2230077E-07	5.6152121E-06	-7.7916804E-06
-5.4670179E-05	2.3516972E-05	3.7524678E-06	-5.1630158E-06	-8.1071614E-08	1.1205048E-07	2.9404605E-06	-1.0726655E-06
-8.5874904E-07	-5.3150412E-05	-2.1385182E-05	-2.5452852E-05	1.0000000E+00	0.	1.3705096E-08	9.6030702E-09
1.0935926E-08	8.1411562E-09	8.7215459E-10	4.5472212E-10	-1.2483932E-06	-6.6508209E-07	3.0333353E-08	8.5534031E-08
-6.2209270E-09	-5.7860143E-09	1.7039495E-10	1.2595871E-10	-3.0201387E-09	-4.6650127E-09	-8.3692082E-08	2.6144962E-09
-3.9576236E-08	3.4282425E-08	-2.3743294E-05	-1.5748330E-03	-5.8094136E-08	4.0261453E-08	-5.8094136E-09	4.0261453E-09
3.2509941E-09	-3.4523510E-10	4.4027230E-08	-3.4627660E-08	-1.1738827E-07	8.0522905E-08	1.6091123E-07	3.3308647E-06
2.3743294E-05	1.5748330E-03	5.1522959E-08	3.4173870E-06				

EIGENVECTOR FOR MODE 20

5.9193268E-07	-6.6430841E-07	-8.4311812E-06	9.0266917E-06	-9.7872040E-07	1.1679650E-06	-5.3718518E-05	5.9424249E-05
1.4934501E-03	-1.6527694E-03	-1.0694287E-06	1.3198735E-06	3.1117191E-08	-3.7924477E-08	1.4198413E-06	-1.4903811E-06
-2.2903244E-06	-3.7405881E-05	-1.8418727E-05	-1.2274044E-05	1.0000000E+00	0.	-9.5727476E-10	-8.0292429E-10
1.2630515E-08	1.1313561E-08	1.6332146E-09	1.3268300E-09	8.3177822E-08	7.2422914E-08	-2.5895627E-06	-2.6231433E-06
1.0452860E-09	1.4764899E-09	-5.3017470E-11	-4.2160832E-11	-2.0986493E-09	-1.9289534E-09	-5.1875672E-09	3.9523227E-09
-1.6560326E-08	2.5669264E-08	-2.1006450E-05	-1.3007367E-03	-3.2243678E-08	2.7603493E-08	-3.2243678E-09	2.7603493E-09
4.0584340E-09	-2.1376263E-10	2.0018761E-08	-2.5980851E-08	-6.4487356E-08	5.5206986E-08	1.1007244E-07	2.9409960E-06
2.1006450E-05	1.3007367E-03	4.2585081E-08	2.9962030E-06				

EIGENVECTOR FOR MODE 21

8.1389186E-08	-6.3660725E-08	-6.2074061E-06	7.7510268E-06	5.5775744E-09	-5.1206498E-09	-2.1826502E-05	1.9330512E-05
3.4015339E-06	-1.5578409E-06	-5.4030524E-08	4.1873531E-08	1.1851911E-09	-9.1379127E-10	4.4447537E-07	-3.8623694E-07
-3.4957209E-07	-1.0734003E-05	-5.2442626E-09	-2.5820278E-06	1.0000000E+00	0.	-8.1303620E-11	-1.0178342E-10
9.8772044E-09	1.0246873E-08	-6.5272939E-12	-6.9654432E-12	2.4650548E-08	2.7266719E-08	-2.0203569E-09	-4.2879794E-09
5.3484733E-11	6.7575086E-11	-1.1672570E-12	-1.4823489E-12	-4.9264275E-10	-5.5535555E-10	-1.3527448E-08	5.7903189E-10
-3.1884855E-09	6.6445570E-09	-1.2887971E-05	-1.2606623E-03	-8.0341016E-09	7.0081124E-09	-8.0341016E-10	7.0081124E-10
3.4304777E-09	-9.0589565E-11	6.0189632E-09	-6.7351460E-09	-1.6068263E-08	1.4016225E-08	4.4035100E-08	2.7216211E-06
1.2887971E-05	1.2606623E-03	2.7966847E-08	2.7356373E-06				

EIGENVECTOR FOR MODE 22

4.4351350E-08	-1.9172637E-08	-2.7522140E-08	1.4961240E-08	1.5047038E-05	-6.3224981E-06	9.2620966E-06	-3.7981014E-06
1.0275181E-05	1.5505845E-06	-3.3674743E-08	1.3048023E-08	7.4723962E-10	-2.9346958E-10	4.8331502E-07	7.6848508E-07
5.6872411E-06	9.9644388E-05	1.8898484E-05	-1.1108016E-05	1.0000000E+00	0.	-1.2649686E-11	-2.8195557E-11
9.8412608E-12	1.7466957E-11	-4.1919980E-09	-9.5670109E-09	-2.5204146E-09	-5.8902057E-09	8.3628429E-10	-1.0629991E-06
9.0610019E-12	2.1416920E-11	-1.9505253E-13	-4.7533076E-13	4.8706394E-10	-3.1674613E-10	6.3710080E-08	-4.6006384E-09
-7.2921527E-09	-1.1985837E-08	-9.7065082E-06	-6.3992776E-04	1.6206080E-08	-3.0051941E-08	1.6206806E-09	-3.0051941E-09
9.2797156E-10	-9.8115061E-12	8.2201243E-09	1.1979025E-08	3.2412013E-08	-6.1303681E-08	-1.1348543E-08	1.4449471E-06
9.7066082E-06	6.3992776E-04	2.1063478E-08	1.3880432E-06				

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 4

-4.2510318E-04	0.	-1.1180743E-04	0.	8.0287040E-05	0.	3.3295730E-04	0.
5.9051830E-04	0.	8.5515662E-04	0.	1.1231325E-03	0.	1.3868818E-03	0.
1.6416701E-03	0.	1.8697795E-03	0.	-1.9780375E-01	0.	1.0000000E+00	0.
-1.9781981E-01	0.	9.9998474E-01	0.				

DEFLECTIONS FOR MODE 5

-2.9764212E-04	0.	-1.1508711E-04	0.	2.9602350E-05	0.	1.7004490E-04	0.
4.0541519E-04	0.	7.9153227E-04	0.	1.2481852E-03	0.	1.6042611E-03	0.
1.7515126E-03	0.	1.7436393E-03	0.	-1.9806931E-01	0.	1.0000000E+00	0.
-1.9830233E-01	0.	9.9970697E-01	0.				

DEFLECTIONS FOR MODE 6

-2.9221204E-04	0.	-1.1205913E-04	0.	2.7452556E-05	0.	1.6157067E-04	0.
3.5538967E-04	0.	7.0041096E-04	0.	1.2580775E-03	0.	1.6173265E-03	0.
1.7575621E-03	0.	1.7333119E-03	0.	-1.9608483E-01	0.	1.0000000E+00	0.
-1.9839153E-01	0.	9.9969330E-01	0.				

DEFLECTIONS FOR MODE 7

-2.0001240E-04	0.	2.1922002E-04	0.	1.4240127E-04	0.	-3.3226077E-04	0.
-4.4026540E-04	0.	3.9905940E-04	0.	1.8339516E-03	0.	2.7282323E-03	0.
2.2610142E-03	0.	7.9698803E-04	0.	-1.9901697E-01	0.	1.0000000E+00	0.
-1.9974242E-01	0.	9.9527456E-01	0.				

DEFLECTIONS FOR MODE 16

-2.9932253E-04	-1.9446770E-04	-1.4174000E-04	-7.6846497E-05	2.6597473E-05	9.4100299E-05	2.4457700E-04	2.7608925E-04
5.1276493E-04	2.9391477E-04	8.1693961E-04	6.6790347E-05	1.1285545E-03	-2.5774143E-04	1.4207634E-03	-3.9843157E-04
1.6830107E-03	-1.7715501E-04	1.9277936E-03	3.0974605E-04	-1.9789305E-01	5.5023598E-04	1.0000000E+00	0.
-1.9813420E-01	1.0880535E-03	9.9975685E-01	5.3781755E-04				

DEFLECTIONS FOR MODE 17

-1.7739614E-01	-1.4375265E-01	-7.3491941E-02	-5.9576003E-02	2.0067229E-02	1.6835050E-02	9.3779564E-02	7.5810041E-02
1.3415795E-01	1.0843535E-01	1.3540244E-01	1.0935098E-01	9.7445942E-02	7.8430250E-02	2.6869557E-02	2.0914143E-02
-6.5297632E-02	-5.4333091E-02	-1.6708249E-01	-1.3752247E-01	-3.9319513E-01	-1.5904640E-01	1.0000000E+00	0.
-4.2441069E-01	-1.8455480E-01	9.6078444E-01	-2.5500404E-02				

DEFLECTIONS FOR MODE 18

-4.4091635E-05	-2.3330462E-04	-1.4337457E-04	-1.8047847E-04	-1.6914598E-04	8.9444906E-05	-3.3653240E-05	4.0517894E-04
3.2525526E-04	6.0210775E-04	8.5940214E-04	1.6973106E-04	1.3900878E-03	-5.3950332E-04	1.7345528E-03	-8.6299411E-04
1.7932806E-03	-3.0550250E-04	1.6072455E-03	7.4827923E-04	-1.9851754E-01	8.8048795E-04	1.0000000E+00	0.
-1.9945583E-01	1.4199841E-03	9.9900171E-01	5.3349610E-04				

DEFLECTIONS FOR MODE 19

```
-1.4142365E-02 -1.3302831E-04   -1.5588720E-04  6.1161011E-05    1.0276958E-02  1.6919720E-04    1.3866239E-02  1.3812657E-04
 9.5971927E-03 -6.1008891E-06   -1.3346438E-04 -1.6777134E-04   -1.0510084E-02 -2.3982803E-04   -1.7002961E-02 -1.6415875E-04
-1.8058357E-02  4.5369714E-05   -1.5613148E-02  3.2551607E-04    3.3243175E-01  5.5621974E-03    2.4259795E-01 -4.2915778E-02
 1.0000000E+00  0.               9.1016620E-01 -5.8477976E-02
```

DEFLECTIONS FOR MODE 20

```
-4.8537474E-03 -3.3444954E-04    2.6676173E-04  4.5796541E-05    4.0861474E-03  3.0353762E-04    5.2285017E-03  3.4005716E-04
 3.1717221E-03  1.5294651E-04   -8.6300697E-04 -1.3811447E-04   -4.1073731E-03 -3.4588534E-04   -3.8038637E-03 -3.2893294E-04
 8.8141505E-04 -3.9763520E-05    8.3197604E-01  3.9311614E-04   -2.0561447E-01 -4.2108259E-04    1.0000000E+00  0.
-2.2817404E-01 -1.7135933E-03    9.7744043E-01 -1.2917107E-03
```

DEFLECTIONS FOR MODE 21

```
 5.1626252E-01 -2.6721171E-02   -7.8749741E-02  4.0140036E-03   -4.7111046E-01  2.[illegible]15938E-02   -5.1101417E-01  2.6448275E-02
-2.1408962E-01  1.1216778E-02    2.1849635E-01 -1.1153428E-02    5.1196494E-01 -2.[illegible]446573E-02    4.6755472E-01 -2.4452331E-02
 7.0371378E-02 -4.3725170E-03   -5.2922075E-01  2.6092455E-02    1.0691844E-01  1.7304161E-02   -2.1485701E-01 -2.3392117E-02
 1.0000000E+00  0.               6.7822455E-01 -4.0780278E-02
```

DEFLECTIONS FOR MODE 22

```
-8.0303467E-01  3.6475261E-04    6.5571232E-01 -2.2422098E-04    9.9979098E-01 -4.3563346E-04    1.0652764E-01 -1.7100166E-05
-9.4709870E-01  2.5778346E-04   -9.4795958E-01  4.3599540E-04    1.0496078E-01  2.6104057E-04    1.0000000E+00  0.
 6.4929304E-01 -4.6200976E-05   -7.9258944E-01  1.3672414E-04   -2.1083505E-01 -1.9768036E-02   -1.5067439E-01  1.0953577E-01
 1.9755399E-01 -1.6359474E-02    2.5771404E-01  1.1294433E-01
```

MINIMUM BLANK COMMON LENGTH REQUIRED = 6053.
BASED ON INPUT DATA AND ANALYSES REQUESTED.

Aerodynamic Influence Coefficients

The aerodynamic influence coefficients (AIC's) at supersonic speeds are found from Piston Theory and are generated by Program PISTON (Ref. 4). Three Mach numbers are considered, 2.0, 3.0, and 4.0, and four reduced velocities are considered, 1000.0, 20.0, 5.0, and 2.0 based on the wing semichord of 1.5 ft. The AIC's for both the wing and the tail (flipper) are found for the same reduced velocities by using the wing semichord as the reference for both surfaces.

The wing planform is shown in Fig. 3 and its motion is determined by the deflections of mass points Nos. 7, 8, and 9 which are located at the fractional wing chord positions 0.152778, 0.569444, and 0.986111, respectively. One strip is sufficient to represent the wing since it has been assumed rigid in the spanwise direction. The basic Piston Theory is used without thickness (a small thickness ratio $\tau = 0.001$ is input). The input data cards are listed below with comments and follow the input format described in the Program PISTON User's Manual (Ref. 4). The input data are also echoed in the program output which follows the input data card listing below. At the end of the printed output the punched card output is listed. The punched AIC's are input appropriately according to the proper partitioning format into Program MPASES.

The tail (flipper) planform is also shown in Fig. 4, and its motion is determined by the deflections of its four mass points, Nos. 11, 12, 13, and 14 which are located at the tail quarter- or

three-quarter chord positions. The tail has been assumed rigid in both spanwise and chordwise directions but to bend and twist about the actuator at the root. Two equal width strips are used to account for the bending motion. Except for the chordwise and spanwise locations of the aerodynamic control points, the tail is treated the same as the wing in terms of input to Program PISTON. The input data cards are listed below those for the wing in the following pages and the printed output of AIC's for the tail follows that for the wing. Again, at the end of the printed output the punched AIC's are listed.

Table 5 - Input Cards for Program PISTON

```
C     TITLE AND SUBTITLE CARDS
 PISTON THEORY AERODYNAMICS FOR WING
 NEW AND IMPROVED MAC MISSILE
C     CONTROL CARDS
    0    1    2    1    1
    1    3    0    4    4    4
C     PLANFORM GEOMETRY
 0.0          1.5          1.0          3.0          3.0
C     STRIP GEOMETRY
 1.0          1.5          0.152777778  0.569444444  0.986111111 0.4
C     THICKNESS DATA
 0.001        0.0          0.0
C     SERIES OF MACH NUMBERS
 2.0          3.0          4.0
C     TRIM ANGLE OF ATTACK FOR EACH MACH NUMBER
 0.0
 0.0
 0.0
C     SERIES OF REDUCED VELOCITIES FOR EACH MACH NUMBER
 1000.0       20.0         5.0          2.0
 1000.0       20.0         5.0          2.0
 1000.0       20.0         5.0          2.0

C     TITLE AND SUBTITLE CARDS
 PISTON THEORY AERODYNAMICS FOR TAIL
 NEW MAC MISSILE
C     CONTROL CARDS
    0    0    2    1    1
    2    3    0    4    4    4
C     PLANFORM GEOMETRY
 0.0          1.5          1.0          3.0          3.0
C     STRIP GEOMETRY
 0.5          0.66666667   0.25         0.50         0.75         0.4
 0.5          0.66666667   0.25         0.50         0.75         0.4
C     THICKNESS DATA
 0.001        0.0          0.0
C     SERIES OF MACH NUMBERS
 2.0          3.0          4.0
C     TRIM ANGLE OF ATTACK FOR EACH MACH NUMBER
 0.0
 0.0
 0.0
C     SERIES OF REDUCED VELOCITIES FOR EACH MACH NUMBER
 1000.0       20.0         5.0          2.0
 1000.0       20.0         5.0          2.0
 1000.0       20.0         5.0          2.0
```

Table 6 - Output from Program PISTON

```
PISTON THEORY AERODYNAMICS FOR WING

NEW AND IMPROVED MAC MISSILE


              AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

                       THICKNESS INTEGRALS CALCULATED FOR AIRFOIL 2

                                       INPUT DATA

                              1 STRIPS
                              3 MACH NUMBERS
                             12 REDUCED FREQUENCIES (TOTAL)

                              SECANT LAMBDA =  0.
                    REFERENCE SEMI-CHORD =   .150000E+01
                                SEMI-SPAN =   .100000E+01
                             SURFACE AREA =   .300000E+01
                                    C BAR =   .300000E+01

  STRIP NO.       DELTA Y          B             Z1             Z2             Z3             ZMAX

      1         .100000E+01    .150000E+01    .152778E+00    .569444E+00    .986111E+00    .400000E+00

  STRIP NO.        TAU          TAU(M)         TAU(T)

      1         .100000E-02    0.             0.

                               MACH NUMBER =      2.000000

                                  1/K(R) =   .100000E+04
                                  1/K(R) =   .200000E+02
                                  1/K(R) =   .500000E+01
                                  1/K(R) =   .200000E+01

                         STRIP NO.         ALPHA ZERO (DEGREES)

                             1                    0.00

                               MACH NUMBER =      3.000000

                                  1/K(R) =   .100000E+04
                                  1/K(R) =   .200000E+02
                                  1/K(R) =   .500000E+01
                                  1/K(R) =   .200000E+01

                         STRIP NO.         ALPHA ZERO (DEGREES)

                             1                    0.00

                               MACH NUMBER =      4.000000

                                  1/K(R) =   .100000E+04
                                  1/K(R) =   .200000E+02
                                  1/K(R) =   .500000E+01
                                  1/K(R) =   .200000E+01

                         STRIP NO.         ALPHA ZERO (DEGREES)

                             1                    0.00
```

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 2,000000

1/K(R) = ,100000E+04

1 STRIPS

CH(1) SIZE = 3 BY 3

,13126448E+07	-,74933925E+03	-,18126021E+07	,66239441E+02	,49995734E+06	,58819818E+01
,80239495E+06	,66239441E+02	,37445451E+06	-,95926251E+03	-,77684946E+06	-,89723991E+02
-,11042446E+06	,58819818E+01	,62892598E+06	-,89723991E+02	-,51850062E+06	-,25619977E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 2,000000

1/K(R) = ,200000E+02

1 STRIPS

CH(1) SIZE = 3 BY 3

,52505790E+03	-,14986785E+02	-,72504084E+03	,13247888E+01	,19998294E+03	,11763964E+00
,16095798E+03	,13247888E+01	,14978180E+03	-,19185250E+02	-,31073979E+03	-,17944798E+01
-,44169782E+02	,11763964E+00	,25157003E+03	-,17944798E+01	-,20740025E+03	-,51239955E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 2,000000

1/K(R) = ,500000E+01

1 STRIPS

CH(1) SIZE = 3 BY 3

,32816219E+02	-,37466963E+01	-,49315052E+02	,33119721E+00	,12498933E+02	,29409909E-01
,10059874E+02	,33119721E+00	,93613628E+01	-,47963125E+01	-,19421237E+02	-,44861996E+00
-,27606114E+01	,29409909E-01	,19723127E+02	-,44861996E+00	-,12962516E+02	-,12809989E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 2,000000

1/K(R) = ,200000E+01

1 STRIPS

CH(1) SIZE = 3 BY 3

,52505790E+01	-,14986785E+01	-,72504084E+01	,13247888E+00	,19998294E+01	,11763964E-01
,16095798E+01	,13247888E+00	,14978180E+01	-,19185250E+01	-,31073979E+01	-,17944798E+00
-,44169782E+00	,11763964E-01	,25157003E+01	-,17944798E+00	-,20740025E+01	-,51239955E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .100000E+04

1 STRIPS

CH(1) SIZE = 3 BY 3

.87679779E+06	-.50055253E+03	-.12100831E+07	.44303712E+02	.33388535E+06	.38543149E+01
.26787508E+06	.44303712E+02	.24971650E+06	-.63926842E+03	-.51759158E+06	-.59655906E+02
-.73387325E+05	.38543149E+01	.41840755E+06	-.59655906E+02	-.34502022E+06	-.17052603E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .200000E+02

1 STRIPS

CH(1) SIZE = 3 BY 3

.35071912E+03	-.10011051E+02	-.48427326E+03	.88607424E+00	.13355418E+03	.77086299E-01
.10715003E+03	.88607424E+00	.99886599E+02	-.12789368E+02	-.20703663E+03	-.11931181E+01
-.29354930E+02	.77086299E-01	.16736302E+03	-.11931181E+01	-.13800809E+03	-.34105325E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .500000E+01

1 STRIPS

CH(1) SIZE = 3 BY 3

.21919945E+02	-.25027627E+01	-.30267078E+02	.22151856E+00	.83471337E+01	.19271575E-01
.66908770E+01	.22151856E+00	.62429124E+01	-.31963421E+01	-.12939789E+02	-.29827953E+00
-.18346831E+01	.19271575E-01	.10460189E+02	-.29827953E+00	-.86255056E+01	-.85263313E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .200000E+01

1 STRIPS

CH(1) SIZE = 3 BY 3

.35071912E+01	-.10011051E+01	-.48427326E+01	.88607424E-01	.13355418E+01	.77086299E-02
.10715003E+01	.88607424E-01	.99886599E+00	-.12789368E+01	-.20703663E+01	-.11931181E+00
-.29354930E+00	.77086299E-02	.16736302E+01	-.11931181E+00	-.13800809E+01	-.34105325E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .100000E+04

1 STRIPS

CH(1) SIZE = 3 BY 3

.65807675E+06	-.37614320E+03	-.96972714E+00	.33314115E+02	.25085039E+06	.28504958E+01
.20061447E+06	.33314115E+02	.18734085E+06	-.47025302E+03	-.38796332E+06	-.44629343E+02
-.54668713E+05	.28504958E+01	.31314920E+06	-.44629343E+02	-.25828057E+06	-.12768765E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .200000E+02

1 STRIPS

CH(1) SIZE = 3 BY 3

.26355070E+03	-.75228640E+01	-.36389086E+03	.66628229E+00	.10034016E+03	.57009916E-01
.80245787E+02	.66628229E+00	.74939541E+02	-.95850603E+01	-.15518533E+03	-.89258687E+00
-.21947485E+02	.57009916E-01	.12525971E+03	-.89258687E+00	-.10331223E+03	-.25537530E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .500000E+01

1 STRIPS

CH(1) SIZE = 3 BY 3

.16471919E+02	-.18807160E+01	-.22743179E+02	.16657057E+00	.62712598E+01	.14252479E-01
.50153617E+01	.16657057E+00	.44837213E+01	-.23962651E+01	-.96990830E+01	-.22314672E+00
-.13717178E+01	.14252479E-01	.78287320E+01	-.22314672E+00	-.64570141E+01	-.63443826E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITH CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .200000E+01

1 STRIPS

CH(1) SIZE = 3 BY 3

.26355070E+01	-.75228640E+00	-.36389086E+01	.66628229E-01	.10034016E+01	.57009916E-02
.80245787E+00	.66628229E-01	.74939541E+00	-.95850603E+00	-.15518533E+01	-.89258687E-01
-.21947485E+00	.57009916E-02	.12525971E+01	-.89258687E-01	-.10331223E+01	-.25537530E+00

```
 1,00000E+03                                                                          PTN 1001
   3   1   1   1                                                                      PTN 1002
   3                                                                                  PTN 1 11
 1,31264E+06-7,49339E+02-1,81260E+06 6,62394E+01 4,99957E+05 5,88198E+00PTN 1 12
 4,02395E+05 6,62394E+01 3,74455E+05-9,59263E+02-7,76849E+05-8,97240E+01PTN 1 13
-1,10428E+05 5,88198E+00 6,28925E+05-8,97240E+01-5,18501E+05-2,56200E+02PTN 1 14
 2,00000E+01                                                                          PTN 2001
   3   1   1   1                                                                      PTN 2002
   3                                                                                  PTN 2 11
 5,25058E+02-1,49868E+01-7,25041E+02 1,32479E+00 1,99983E+02 1,17640E-01PTN 2 12
 1,60958E+02 1,32479E+00 1,49782E+02-1,91853E+01-3,10740E+02-1,79448E+00PTN 2 13
-4,41698E+01 1,17640E-01 2,51570E+02-1,79448E+00-2,07400E+02-5,12400E+00PTN 2 14
 5,00000E+00                                                                          PTN 3001
   3   1   1   1                                                                      PTN 3002
   3                                                                                  PTN 3 11
 3,28141E+01-3,74670E+00-4,53151E+01 3,31197E-01 1,24989E+01 2,94099E-02PTN 3 12
 1,00599E+01 3,31197E-01 9,36136E+00-4,79631E+00-1,94212E+01-4,48620E-01PTN 3 13
-2,76061E+00 2,94099E-02 1,57231E+01-4,48620E-01-1,29625E+01-1,28100E+00PTN 3 14
 2,00000E+00                                                                          PTN 4001
   3   1   1   1                                                                      PTN 4002
   3                                                                                  PTN 4 11
 5,25058E+00-1,49868E+00-7,25041E+00 1,32479E-01 1,99983E+00 1,17640E-02PTN 4 12
 1,60958E+00 1,32479E-01 1,49782E+00-1,91853E+00-3,10740E+00-1,79448E-01PTN 4 13
-4,41698E-01 1,17640E-02 2,51570E+00-1,79448E-01-2,07400E+00-5,12400E-01PTN 4 14
 1,00000E+03                                                                          PTN 1001
   3   1   1   1                                                                      PTN 1002
   3                                                                                  PTN 1 11
 8,76798E+05-5,00553E+02-1,21068E+06 4,43037E+01 3,33885E+05 3,85431E+00PTN 1 12
 2,67875E+05 4,43037E+01 2,49716E+05-6,39268E+02-5,17592E+05-5,96559E+01PTN 1 13
-7,33873E+04 3,85431E+00 4,18408E+05-5,96559E+01-3,45020E+05-1,70527E+02PTN 1 14
 2,00000E+01                                                                          PTN 2001
   3   1   1   1                                                                      PTN 2002
   3                                                                                  PTN 2 11
 3,50719E+02-1,00111E+01-4,84273E+02 8,86074E-01 1,33554E+02 7,70863E-02PTN 2 12
 1,07150E+02 8,86074E-01 9,98866E+01-1,27854E+01-2,07037E+02-1,19312E+00PTN 2 13
-2,93549E+01 7,70863E-02 1,67363E+02-1,19312E+00-1,38008E+02-3,41053E+00PTN 2 14
 5,00000E+00                                                                          PTM 3001
   3   1   1   1                                                                      PTN 3002
   3                                                                                  PTN 3 11
 2,19199E+01-2,50276E+00-3,02671E+01 2,21519E-01 8,34713E+00 1,92716E-02PTN 3 12
 6,69688E+00 2,21519E-01 6,24291E+00-3,19634E+00-1,29398E+01-2,98280E-01PTN 3 13
-1,83468E+00 1,92716E-02 1,04602E+01-2,98280E-01-8,62551E+00-8,52633E-01PTN 3 14
 2,00000E+00                                                                          PTN 4001
   3   1   1   1                                                                      PTN 4002
   3                                                                                  PTN 4 11
 3,50719E+00-1,00111E+00-4,84273E+00 8,86074E-02 1,33554E+00 7,70863E-03PTN 4 12
 1,07150E+00 8,86074E-02 9,98866E-01-1,27854E+00-2,07037E+00-1,19312E-01PTN 4 13
-2,93549E-01 7,70863E-03 1,67363E+00-1,19312E-01-1,38008E+00-3,41053E-01PTN 4 14
 1,00000E+03                                                                          PTN 1001
   3   1   1   1                                                                      PTN 1002
   3                                                                                  PTN 1 11
 6,58877E+05-3,76143E+02-9,09727E+05 3,33141E+01 2,50850E+05 2,85050E+00PTN 1 12
 2,00614E+05 3,33141E+01 1,87349E+05-4,79253E+02-3,87963E+05-4,46293E+01PTN 1 13
-5,48487E+04 2,85050E+00 3,13149E+05-4,46293E+01-2,58281E+05-1,27688E+02PTN 1 14
 2,00000E+01                                                                          PTN 2001
   3   1   1   1                                                                      PTN 2002
   3                                                                                  PTM 2 11
 2,63551E+02-7,52286E+00-3,63891E+02 6,66282E-01 1,00340E+02 5,70099E-02PTN 2 12
 8,02458E+01 6,66282E-01 7,49395E+01-9,58506E+00-1,55185E+02-8,92587E-01PTN 2 13
-2,19475E+01 5,70099E-02 1,25260E+02-8,92587E-01-1,03312E+02-2,55375E+00PTN 2 14
 5,00000E+00                                                                          PTN 3001
   3   1   1   1                                                                      PTN 3002
   3                                                                                  PTN 3 11
 1,64719E+01-1,88072E+00-2,27432E+01 1,66571E-01 6,27126E+00 1,42525E-02PTN 3 12
 5,01536E+00 1,66571E-01 4,68372E+00-2,39627E+00-9,69906E+00-2,23147E-01PTN 3 13
-1,37172E+00 1,42525E-02 7,82873E+00-2,23147E-01-6,45701E+00-6,38438E-01PTN 3 14
 2,00000E+00                                                                          PTN 4001
   3   1   1   1                                                                      PTN 4002
   3                                                                                  PTN 4 11
 2,63551E+00-7,52286E-01-3,63891E+00 6,66282E-02 1,00340E+00 5,70099E-03PTN 4 12
 8,02458E-01 6,66282E-02 7,49395E-01-9,58506E-01-1,55185E+00-8,92587E-02PTN 4 13
-2,19475E-01 5,70099E-03 1,25260E+00-8,92587E-02-1,03312E+00-2,55375E-01PTN 4 14
```

[illegible] THEORY AERODYNAMICS FOR TAIL

[illegible] MAG MISSILE

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

THICKNESS INTEGRALS CALCULATED FOR AIRFOIL 2

INPUT DATA

2 STRIPS
3 MACH NUMBERS
12 REDUCED FREQUENCIES (TOTAL)

SECANT LAMBDA = 0.
REFERENCE SEMI-CHORD = .150000E+01
SEMI-SPAN = .100000E+01
SURFACE AREA = .300000E+01
C BAR = .300000E+01

STRIP NO.	DELTA Y	B	Z1	Z2	Z3	ZMAX
1	.500000E+00	.666667E+00	.250000E+00	.500000E+00	.750000E+00	.400000E+00
2	.500000E+00	.666667E+00	.250000E+00	.500000E+00	.750000E+00	.400000E+00

STRIP NO.	TAU	TAU(M)	TAU(T)
1	.100000E-02	0.	0.
2	.100000E-02	0.	0.

MACH NUMBER = 2.000000

1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO.	ALPHA ZERO (DEGREES)
1	0.00
2	0.00

MACH NUMBER = 3.000000

1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO.	ALPHA ZERO (DEGREES)
1	0.00
2	0.00

MACH NUMBER = 4.000000

1/K(R) = .100000E+04
1/K(R) = .200000E+02
1/K(R) = .500000E+01
1/K(R) = .200000E+01

STRIP NO.	ALPHA ZERO (DEGREES)
1	0.00
2	0.00

```
AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER
OSCILLATORY CASE
MACH NO. =      2.000000
1/K(R) =   .100000E+04
2 STRIPS
CH( 1) SIZE = 2 BY 2

 .50160233E+06  -.26004306E+03  -.50160233E+06   .37108691E+02
 .49840100E+06   .37108691E+02  -.49840100E+06  -.25862025E+03

CH( 2) SIZE = 2 BY 2

 .50160233E+06  -.26004306E+03  -.50160233E+06   .37108691E+02
 .49840100E+06   .37108691E+02  -.49840100E+06  -.25862025E+03
```

```
AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER
OSCILLATORY CASE
MACH NO. =      2.000000
1/K(R) =   .200000E+02
2 STRIPS
CH( 1) SIZE = 2 BY 2

 .20064093E+03  -.52008612E+01  -.20064093E+03   .74217381E+00
 .19936040E+03   .74217381E+00  -.19936040E+03  -.51724049E+01

CH( 2) SIZE = 2 BY 2

 .20064093E+03  -.52008612E+01  -.20064093E+03   .74217381E+00
 .19936040E+03   .74217381E+00  -.19936040E+03  -.51724049E+01
```

```
AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER
OSCILLATORY CASE
MACH NO. =      2.000000
1/K(R) =   .500000E+01
2 STRIPS
CH( 1) SIZE = 2 BY 2

 .12540058E+02  -.13002153E+01  -.12540058E+02   .18554345E+00
 .12460025E+02   .18554345E+00  -.12460025E+02  -.12931012E+01

CH( 2) SIZE = 2 BY 2

 .12540058E+02  -.13002153E+01  -.12540058E+02   .18554345E+00
 .12460025E+02   .18554345E+00  -.12460025E+02  -.12931012E+01
```

```
AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER
OSCILLATORY CASE
MACH NO. =      2.000000
1/K(R) =   .200000E+01
2 STRIPS
CH( 1) SIZE = 2 BY 2

 .20064093E+01  -.52008612E+00  -.20064093E+01   .74217381E-01
 .19936040E+01   .74217381E-01  -.19936040E+01  -.51724049E+00

CH( 2) SIZE = 2 BY 2

 .20064093E+01  -.52008612E+00  -.20064093E+01   .74217381E-01
 .19936040E+01   .74217381E-01  -.19936040E+01  -.51724049E+00
```

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .100000E+04

2 STRIPS

CM(1) SIZE = 2 BY 2

.33493683E+06	-.17362810E+03	-.33493683E+06	.24763283E+02
.33173483E+06	.24763283E+02	-.33173483E+06	-.17220099E+03

CM(2) SIZE = 2 BY 2

.33493683E+06	-.17362810E+03	-.33493683E+06	.24763283E+02
.33173483E+06	.24763283E+02	-.33173483E+06	-.17220099E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .200000E+02

2 STRIPS

CM(1) SIZE = 2 BY 2

.13397473E+03	-.34724820E+01	-.13397473E+03	.49526565E+00
.13269393E+03	.49526565E+00	-.13269393E+03	-.34440197E+01

CM(2) SIZE = 2 BY 2

.13397473E+03	-.34724820E+01	-.13397473E+03	.49526565E+00
.13269393E+03	.49526565E+00	-.13269393E+03	-.34440197E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .500000E+01

2 STRIPS

CM(1) SIZE = 2 BY 2

.83734208E+01	-.86812049E+00	-.83734208E+01	.12381641E+00
.82933708E+01	.12381641E+00	-.82933708E+01	-.86100494E+00

CM(2) SIZE = 2 BY 2

.83734208E+01	-.86812049E+00	-.83734208E+01	.12381641E+00
.82933708E+01	.12381641E+00	-.82933708E+01	-.86100494E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 3.000000

1/K(R) = .200000E+01

2 STRIPS

CM(1) SIZE = 2 BY 2

.13397473E+01	-.34724820E+00	-.13397473E+01	.49526565E-01
.13269393E+01	.49526565E-01	-.13269393E+01	-.34440197E+00

CM(2) SIZE = 2 BY 2

.13397473E+01	-.34724820E+00	-.13397473E+01	.49526565E-01
.13269393E+01	.49526565E-01	-.13269393E+01	-.34440197E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .100000E+04

2 STRIPS

CH(1) SIZE = 2 BY 2

.25160467E+06	-.13041501E+03	-.25160487E+06	.18590714E+02
.24840200E+06	.18590714E+02	-.24840200E+06	-.12849160E+03

CH(2) SIZE = 2 BY 2

.25160467E+06	-.13041501E+03	-.25160467E+06	.18590714E+02
.24840206E+06	.18590714E+02	-.24840200E+06	-.12849160E+03

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .200000E+02

2 STRIPS

CH(1) SIZE = 2 BY 2

.10064187E+03	-.26083002E+01	-.10064187E+03	.37181428E+00
.99360800E+02	.37181428E+00	-.99360800E+02	-.25798321F+01

CH(2) SIZE = 2 BY 2

.10064187E+03	-.26083002E+01	-.10064187E+03	.37181428F+00
.99360800E+02	.37181428E+00	-.99360860E+02	-.25798321E+01

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .500000E+01

2 STRIPS

CH(1) SIZE = 2 BY 2

.62901167E+01	-.65207506E+00	-.62901187E+01	.92953571E-01
.62100500E+01	.92953571E-01	-.62100500E+01	-.64495802E+00

CH(2) SIZE = 2 BY 2

.62901167E+01	-.65207506E+00	-.62901167E+01	.92953571E-01
.62100500E+01	.92953571E-01	-.62100500E+01	-.64495802E+00

AERODYNAMIC INFLUENCE COEFFICIENTS BY PISTON THEORY WITHOUT CAMBER

OSCILLATORY CASE

MACH NO. = 4.000000

1/K(R) = .200000E+01

2 STRIPS

CH(1) SIZE = 2 BY 2

.10064187E+01	-.26083002E+00	-.10064187E+01	.37181428E-01
.99360800E+00	.37181428E-01	-.99360800E+00	-.25798321E+00

CH(2) SIZE = 2 BY 2

.10064187E+01	-.26083002E+00	-.10064187E+01	.37181428E-01
.99360800E+00	.37181428E-01	-.99360800E+00	-.25798321E+00

```
1.00000E+03                                                PTN 1001
   4   2   1   1                                           PTN 1002
   2                                                       PTN 1 11
5.01602E+05-2.60043E+02-5.01602E+05 3.71087E+01            PTN 1 12
4.98401E+05 3.71087E+01-4.98401E+05-2.58620E+02            PTN 1 12
   2                                                       PTN 1 21
5.01602E+05-2.60043E+02-5.01602E+05 3.71087E+01            PTN 1 22
4.98401E+05 3.71087E+01-4.98401E+05-2.58620E+02            PTN 1 22
2.00000E+01                                                PTN 2001
   4   2   1   1                                           PTN 2002
   2                                                       PTN 2 11
2.00641E+02-5.20086E+00-2.00641E+02 7.42174E-01            PTN 2 12
1.99360E+02 7.42174E-01-1.99360E+02-5.17240E+00            PTN 2 12
   2                                                       PTN 2 21
2.00641E+02-5.20086E+00-2.00641E+02 7.42174E-01            PTN 2 22
1.99360E+02 7.42174E-01-1.99360E+02-5.17240E+00            PTN 2 22
5.00000E+00                                                PTN 3001
   4   2   1   1                                           PTN 3002
   2                                                       PTN 3 11
1.25401E+01-1.30022E+00-1.25401E+01 1.85543E-01            PTN 3 12
1.24600E+01 1.85543E-01-1.24600E+01-1.29310E+00            PTN 3 12
   2                                                       PTN 3 21
1.25401E+01-1.30022E+00-1.25401E+01 1.85543E-01            PTN 3 22
1.24600E+01 1.85543E-01-1.24600E+01-1.29310E+00            PTN 3 22
2.00000E+00                                                PTN 4001
   4   2   1   1                                           PTN 4002
   2                                                       PTN 4 11
2.00641E+00-5.20086E-01-2.00641E+00 7.42174E-02            PTN 4 12
1.99360E+00 7.42174E-02-1.99360E+00-5.17240E-01            PTN 4 12
   2                                                       PTN 4 21
2.00641E+00-5.20086E-01-2.00641E+00 7.42174E-02            PTN 4 22
1.99360E+00 7.42174E-02-1.99360E+00-5.17240E-01            PTN 4 22
1.00000E+03                                                PTN 1001
   4   2   1   1                                           PTN 1002
   2                                                       PTN 1 11
3.34937E+05-1.73628E+02-3.34937E+05 2.47633E+01            PTN 1 12
3.31735E+05 2.47633E+01-3.31735E+05-1.72201E+02            PTN 1 12
   2                                                       PTN 1 21
3.34937E+05-1.73628E+02-3.34937E+05 2.47633E+01            PTN 1 22
3.31735E+05 2.47633E+01-3.31735E+05-1.72201E+02            PTN 1 22
2.00000E+01                                                PTN 2001
   4   2   1   1                                           PTN 2002
   2                                                       PTN 2 11
1.33975E+02-3.47248E+00-1.33975E+02 4.95266E-01            PTN 2 12
1.32694E+02 4.95266E-01-1.32694E+02-3.44402E+00            PTN 2 12
   2                                                       PTN 2 21
1.33975E+02-3.47248E+00-1.33975E+02 4.95266E-01            PTN 2 22
1.32694E+02 4.95266E-01-1.32694E+02-3.44402E+00            PTN 2 22
5.00000E+00                                                PTN 3001
   4   2   1   1                                           PTN 3002
   2                                                       PTN 3 11
8.37342E+00-8.68120E-01-8.37342E+00 1.23816E-01            PTN 3 12
8.29337E+00 1.23816E-01-8.29337E+00-8.61005E-01            PTN 3 12
   2                                                       PTN 3 21
8.37342E+00-8.68120E-01-8.37342E+00 1.23816E-01            PTN 3 22
8.29337E+00 1.23816E-01-8.29337E+00-8.61005E-01            PTN 3 22
2.00000E+00                                                PTN 4001
   4   2   1   1                                           PTN 4002
   2                                                       PTN 4 11
1.33975E+00-3.47248E-01-1.33975E+00 4.95266E-02            PTN 4 12
1.32694E+00 4.95266E-02-1.32694E+00-3.44402E-01            PTN 4 12
   2                                                       PTN 4 21
1.33975E+00-3.47248E-01-1.33975E+00 4.95266E-02            PTN 4 22
1.32694E+00 4.95266E-02-1.32694E+00-3.44402E-01            PTN 4 22
1.00000E+03                                                PTN 1001
   4   2   1   1                                           PTN 1002
   2                                                       PTN 1 11
2.51605E+05-1.30415E+02-2.51605E+05 1.85907E+01            PTN 1 12
2.48802E+05 1.85907E+01-2.48802E+05-1.28992E+02            PTN 1 12
   2                                                       PTN 1 21
2.51605E+05-1.30415E+02-2.51605E+05 1.85907E+01            PTN 1 22
2.48802E+05 1.85907E+01-2.48802E+05-1.28992E+02            PTN 1 22
2.00000E+01                                                PTN 2001
   4   2   1   1                                           PTN 2002
   2                                                       PTN 2 11
1.00642E+02-2.60830E+00-1.00642E+02 3.71814E-01            PTN 2 12
9.93608E+01 3.71814E-01-9.93608E+01-2.57983E+00            PTN 2 12
   2                                                       PTN 2 21
1.00642E+02-2.60830E+00-1.00642E+02 3.71814E-01            PTN 2 22
9.93608E+01 3.71814E-01-9.93608E+01-2.57983E+00            PTN 2 22
5.00000E+00                                                PTN 3001
   4   2   1   1                                           PTN 3002
   2                                                       PTN 3 11
6.29012E+00-6.52079E-01-6.29012E+00 9.29536E-02            PTN 3 12
6.21005E+00 9.29536E-02-6.21005E+00-6.44958E-01            PTN 3 12
   2                                                       PTN 3 21
6.29012E+00-6.52079E-01-6.29012E+00 9.29536E-02            PTN 3 22
6.21005E+00 9.29536E-02-6.21005E+00-6.44958E-01            PTN 3 22
2.00000E+00                                                PTN 4001
   4   2   1   1                                           PTN 4002
   2                                                       PTN 4 11
1.00642E+00-2.60830E-01-1.00642E+00 3.71814E-02            PTN 4 12
9.93608E-01 3.71814E-02-9.93608E-01-2.57983E-01            PTN 4 12
   2                                                       PTN 4 21
1.00642E+00-2.60830E-01-1.00642E+00 3.71814E-02            PTN 4 22
9.93608E-01 3.71814E-02-9.93608E-01-2.57983E-01            PTN 4 22
```

Aero-Servo-Elastic Stability Analysis

The input data for the aero-servo-elastic stability analysis are obtained by adding the aerodynamic influence coefficients (AIC's) to the servoelastic stability analysis data deck and by adding the necessary aerodynamic control cards and related data. The complete input data deck is listed in Table 7. The AIC's were based on the three Mach numbers M = 2.0, 3.0, and 4.0. The stability analysis is carried out at sea level for which the density is 0.002378 slugs/cu. ft., and the speed of sound is 1100.0 fps, and at the three Mach numbers 2.5, 3.0, and 3.5; the eigenvectors are requested for M = 3.0.

The results of the analyses at the three Mach numbers are shown in Table 8. Table 8 shows an abbreviated output: it shows the first page which echoes the aerodynamic control data, a typical printout of the AIC's for M = 2.0 and k = 0.001, the three sets of stability results, and the eigenvectors for M = 3.0. There are some differences from the servoelastic results in Table 4. Both the servoelastic analysis and the aero-servo-elastic analysis have 22 roots. However, the 15th root, which was a complex conjugate "zero" in the servoelastic analysis, now has become the short period root in the presence of the airstream. In addition, an unstable root, No. 3, has also appeared. A perusal of the 3rd eigenvector and structural deflections indicates it to be some kind of divergence in altitude. This instability is a subject for further research.

The motions involved in the eight oscillatory and five non-oscillatory solutions are summarized below.

Mode No.	Dominant Motion
3	Altitude divergence
4	Actuator
5 & 6	Accelerometer
7	Noise filter
15	Short period
16	First body bending
17	Rate gyro
18	Compensating damping amplifier
19	Flipper bending
20	Second body bending
21	Flipper torsion
22	Third body bending

Table 7 - Input Cards for Program MPASES for Aero-Servo-Elastic Stability Analysis

```
C      TITLE CARDS
 TYPICAL AIR-TO-AIR MISSILE
 AERO-SERVO-ELASTIC ANALYSIS USING PISTON THEORY
C      CONTROL CARD
    2   14    5    2    1   11    5    3    4    1   30    1    0    1
C      SHIFT EIGENVALUE
 100.0
C      AERO CONSTANTS
 1.0           3.0           3.0           0.05
C      AIC DATA
C      MACH NUMBERS
 2.0           3.0           4.0
C      REDUCED FREQUENCIES
 0.001         0.05          0.20          0.50
C      AERO DATA
 0.002378      1100.0
C      NUMBER OF VELOCITIES
    3
C      MASS MATRIX
C      1ST ROW
 50.0          0.0           0.0           0.0           0.0           0.0
 0.0           0.0           0.0           0.0           0.0           0.0
 0.0           0.0
C      2ND ROW
 50.0          0.00          0.0           0.0           0.0           0.0
 0.0           0.0           0.0           0.0           0.0           0.0
 0.0
C      3RD ROW
 50.0          0.0           0.00          0.0           0.0           0.0
 0.0           0.0           0.0           0.0           0.0           0.0
C      4TH ROW
 50.0          0.0           0.0           0.0           0.0           0.0
 0.0           0.0           0.0           0.0           0.0
C      5TH ROW
 50.0          0.0           0.0           0.0           0.0           0.0
 0.0           0.0           0.0           0.0
C      6TH ROW
 50.0          0.0           0.0           0.0           0.0           0.0
 0.0           0.0           0.0
C      7TH ROW
 50.0          0.0           0.0           0.0           0.0           0.0
 0.0           0.0
C      8TH ROW
 50.0          0.0           0.0           0.0           0.0           0.0
 0.0
C      9TH ROW
 50.0          0.0           0.0           0.0           0.0           0.0
C      10TH ROW
 50.0          0.0           0.0           0.0           0.0
C      11TH ROW
 4.4333333     0.0010417     -0.3961806    0.0
C      12TH ROW
 0.9701389     0.0           -0.7260417
C      13TH ROW
 1.1333332     -0.2961806
C      14TH ROW
 1.2979166
C      CONTROL SURFACE DEFLECTION MODE
 0.0           0.0           0.0           0.0           0.0           0.0
 0.0           0.0           0.0           0.0           1.333333333-6.666666667
 1.333333333-6.666666667
```

```
C        DAMPING COEFFICIENTS, G
 0.01          0.02         0.03         0.03         0.03
C      MODAL VIBRATION FREQUENCIES
 45.0          125.4        248.2        100.0        115.0
C      VIBRATION MODE SHAPES
C      1ST MODE
 1.0           0.411195     -0.122424    -0.532567    -0.756204    -0.756204
-0.532467     -0.122424     0.411195     1.0          0.821620     1.138881
 0.821620      1.138881
C      2ND MODE
 1.0          -0.150871    -0.910842    -0.990216    -0.419043     0.419043
 0.990215      0.910842     0.150871    -1.0         -0.630256    -1.281954
-0.630256     -1.281954
C      3RD MODE
-0.803195      0.645103     1.0          0.105986    -0.947894    -0.947893
 0.105986      1.0          0.645103    -0.803195    -0.259298    -1.188204
-0.259298     -1.188204
C      4TH MODE
 0.0           0.0          0.0          0.0          0.0          0.0
 0.0           0.0          0.0          0.0          .3333333333  .3333333333
 1.0           1.0
C      5TH MODE
 0.0           0.0          0.0          0.0          0.0          0.0
 0.0           0.0          0.0          0.0         -0.2          1.0
-0.2           1.0
C      RIGID BODY MODES
C      PLUNGE MODE
 1.0           1.0          1.0          1.0          1.0          1.0
 1.0           1.0          1.0          1.0          1.0          1.0
 1.0           1.0
C      PITCH MODE
 7.5           22.5         37.5         52.5         67.5         82.5
 97.5          112.5        127.5        142.5        138.0        146.0
 138.0         146.0
C      CONTROL SURFACE OUTPUT
 1.0           0.016        0.0
C      NUMBER OF INPUTS TO CONTROL SURFACE
     1
C      CONTROL SURFACE INPUTS FROM SERVOS
     5    0   -1.0
C      OUTPUT FROM SERVO ELEMENT
     1    2    1.0          0.359     -2 0.659     -5
C      NUMBER OF INPUTS TO SERVO ELEMENT
     1
C      INPUT FROM BODY PITCH RATE
     0   -1   -2.7777778-3
C      DIFFERENTIATION MATRIX FOR RATE GYRO
 0.0           1.0         -27.0         27.0        -1.0          0.0
 0.0           0.0          0.0          0.0          0.0          0.0
 0.0           0.0
C      OUTPUT FROM SERVO ELEMENT
     2    1    1.0          0.001
C      NUMBER OF INPUTS TO SERVO ELEMENT
     1
C      INPUTS TO SERVO ELEMENT
     1    0   -1.0
C      OUTPUT FROM SERVO ELEMENT
     3    0    1.0
C      NUMBER OF INPUTS TO SERVO ELEMENT
     1
C      INPUTS TO SERVO ELEMENT
     2    0   -0.1
C      OUTPUT FROM SERVO ELEMENT
     4    2    1.0          0.223     -2 0.253     -5
C      NUMBER OF INPUTS TO SERVO ELEMENT
     1
```

```
C       INPUTS TO SERVO ELEMENT
    3    1  -1.0            -0.016
C       OUTPUT FROM SERVO ELEMENT
    5    0   1.0
C       NUMBER OF INPUTS TO SERVO ELEMENT
    2
C       INPUTS TO SERVO ELEMENT
    4    0   1.0
    8    0  -1.0
C       OUTPUT FROM SERVO ELEMENT
    6    0   1.0
C       NUMBER OF INPUTS TO SERVO ELEMENT
    1
C       INPUTS TO SERVO ELEMENT
    2    0  -2.0
C       OUTPUT FROM SERVO ELEMENT
    7    0   1.0
C       NUMBER OF INPUTS TO SERVO ELEMENT
    2
C       INPUTS TO SERVO ELEMENT
    6    0   1.0
   11    0  -1.0
C       OUTPUT FROM SERVO ELEMENT
    8    1   0.0             1.0
C       NUMBER OF INPUTS TO SERVO ELEMENT
    1
C       INPUTS TO SERVO ELEMENT
    7    0  -1.0
C       OUTPUT FROM SERVO ELEMENT
    9    2   1.0            0.6366      -2 1.013     -5
C       NUMBER OF INPUTS TO SERVO ELEMENT
    1
C       INPUT FROM BODY ACCELERATION
    0   -2  -0.0625
C       INTERPOLATION MATRIX FOR ACCELEROMETER
 0.0          -1.0          9.0          9.0          -1.0          0.0
 0.0           0.0          0.0          0.0           0.0          0.0
 0.0           0.0
C       OUTPUT FROM SERVO ELEMENT
   10    0   1.0
C       NUMBER OF INPUTS TO SERVO ELEMENT
    1
C       INPUTS TO SERVO ELEMENT
    9    0   1.0
C       OUTPUT FROM SERVO ELEMENT
   11    0   1.0
C       NUMBER OF INPUTS TO SERVO ELEMENT
    1
C       OUTPUT FROM SERVO ELEMENT
   10    0  -0.002170
C       AICS
C       1ST K, 1ST M
C       AIC PARTITION CONTROL
    6    1
C       AIC PARTITION CONTROL
    3    0
C       AIC PARTITION
 1.31264E+06-7.49339E+02-1.81260E+06 6.62394E+01 4.99957E+05 5.88198E+00PTN 1 12
 4.02395E+05 6.62394E+01 3.74455E+05-9.59263E+02-7.76849E+05-8.97240E+01PTN 1 13
-1.10424E+05 5.88198E+00 6.28925E+05-8.97240E+01-5.18501E+05-2.56200E+02PTN 1 14
C       AIC PARTITION CONTROL
    1    1
C       AIC PARTITION CONTROL
    2    0
C       AIC PARTITION
 5.01602E+05-2.60043E+02-5.01602E+05 3.71087E+01                        PTN 1 12
```

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 4,98401E+05 3,71087E+01-4,98401E+05-2,58620E+02                               PTN 1 12
C      AIC PARTITION CONTROL
    2     0
C      AIC PARTITION
 5,01602E+05-2,60043E+02-5,01602E+05 3,71087E+01                               PTN 1 22
 4,98401E+05 3,71087E+01-4,98401E+05-2,58620E+02                               PTN 1 22
C      2ND K, 1ST M
C      AIC PARTITION CONTROL
    6     1
C      AIC PARTITION CONTROL
    3     0
C      AIC PARTITION
 5,25058E+02-1,49868E+01-7,25041E+02 1,32479E+00 1,99983E+02 1,17640E-01PTN 2 12
 1,60958E+02 1,32479E+00 1,49782E+02-1,91853E+01-3,10740E+02-1,79448E+00PTN 2 13
-4,41698E+01 1,17640E-01 2,51570E+02-1,79448E+00-2,07400E+02-5,12400E+00PTN 2 14
C      AIC PARTITION CONTROL
    1     1
C      AIC PARTITION CONTROL
    2     0
C      AIC PARTITION
 2,00641E+02-5,20086E+00-2,00641E+02 7,42174E-01                               PTN 2 12
 1,99360E+02 7,42174E-01-1,99360E+02-5,17240E+00                               PTN 2 12
C      AIC PARTITION CONTROL
    2     0
C      AIC PARTITION
 2,00641E+02-5,20086E+00-2,00641E+02 7,42174E-01                               PTN 2 22
 1,99360E+02 7,42174E-01-1,99360E+02-5,17240E+00                               PTN 2 22
C      3RD K, 1ST M
C      AIC PARTITION CONTROL
    6     1
C      AIC PARTITION CONTROL
    3     0
C      AIC PARTITION
 3,28161E+01-3,74670E+00-4,53151E+01 3,31197E-01 1,24989E+01 2,94099E-02PTN 3 12
 1,00599E+01 3,31197E-01 9,36136E+00-4,79631E+00-1,94212E+01-4,48620E-01PTN 3 13
-2,76061E+00 2,94099E-02 1,57231E+01-4,48620E-01-1,29625E+01-1,28100E+00PTN 3 14
C      AIC PARTITION CONTROL
    1     1
C      AIC PARTITION CONTROL
    2     0
C      AIC PARTITION
 1,25401E+01-1,30022E+00-1,25401E+01 1,85543E-01                               PTN 3 12
 1,24600E+01 1,85543E-01-1,24600E+01-1,29310E+00                               PTN 3 12
C      AIC PARTITION CONTROL
    2     0
C      AIC PARTITION
 1,25401E+01-1,30022E+00-1,25401E+01 1,85543E-01                               PTN 3 22
 1,24600E+01 1,85543E-01-1,24600E+01-1,29310E+00                               PTN 3 22
C      4TH K, 1ST M
C      AIC PARTITION CONTROL
    6     1
C      AIC PARTITION CONTROL
    3     0
C      AIC PARTITION
 5,25058E+00-1,49868E+00-7,25041E+00 1,32479E-01 1,99983E+00 1,17640E-02PTN 4 12
 1,60958E+00 1,32479E-01 1,49782E+00-1,91853E+00-3,10740E+00-1,79448E-01PTN 4 13
-4,41698E-01 1,17640E-02 2,51570E+00-1,79448E-01-2,07400E+00-5,12400E-01PTN 4 14
C      AIC PARTITION CONTROL
    1     1
C      AIC PARTITION CONTROL
    2     0
C      AIC PARTITION
 2,00641E+00-5,20086E-01-2,00641E+00 7,42174E-02                               PTN 4 12
 1,99360E+00 7,42174E-02-1,99360E+00-5,17240E-01                               PTN 4 12
C      AIC PARTITION CONTROL
    2     0
```

```
C       AIC PARTITION
 2.00641E+00-5.20086E-01-2.00641E+00 7.42174E-02                                    PTN 4 22
 1.99360E+00 7.42174E-02-1.99360E+00-5.17240E-01                                    PTN 4 22
C       1ST K, 2ND M
C       AIC PARTITION CONTROL
    6    1
C       AIC PARTITION CONTROL
    3    0
C       AIC PARTITION
 8.76798E+05-5.00553E+02-1.21068E+06 4.43037E+01 3.33885E+05 3.85431E+00PTN 1 12
 2.67875E+05 4.43037E+01 2.49716E+05-6.39268E+02-5.17592E+05-5.96559E+01PTN 1 13
-7.33873E+04 3.85431E+00 4.18408E+05-5.96559E+01-3.45020E+05-1.70527E+02PTN 1 14
C       AIC PARTITION CONTROL
    1    1
C       AIC PARTITION CONTROL
    2    0
C       AIC PARTITION
 3.34937E+05-1.73624E+02-3.34937E+05 2.47633E+01                                    PTN 1 12
 3.31735E+05 2.47633E+01-3.31735E+05-1.72201E+02                                    PTN 1 12
C       AIC PARTITION CONTROL
    2    0
C       AIC PARTITION
 3.34937E+05-1.73624E+02-3.34937E+05 2.47633E+01                                    PTN 1 22
 3.31735E+05 2.47633E+01-3.31735E+05-1.72201E+02                                    PTN 1 22
C       2ND K, 2ND M
C       AIC PARTITION CONTROL
    6    1
C       AIC PARTITION CONTROL
    3    0
C       AIC PARTITION
 3.50719E+02-1.00111E+01-4.84273E+02 8.86074E-01 1.33554E+02 7.70863E-02PTN 2 12
 1.07150E+02 8.86074E-01 9.98866E+01-1.27854E+01-2.07037E+02-1.19312E+00PTN 2 13
-2.93549E+01 7.70863E-02 1.67363E+02-1.19312E+00-1.38008E+02-3.41053E+00PTN 2 14
C       AIC PARTITION CONTROL
    1    1
C       AIC PARTITION CONTROL
    2    0
C       AIC PARTITION
 1.33975E+02-3.47248E+00-1.33975E+02 4.95266E-01                                    PTN 2 12
 1.32694E+02 4.95266E-01-1.32694E+02-3.44402E+00                                    PTN 2 12
C       AIC PARTITION CONTROL
    2    0
C       AIC PARTITION
 1.33975E+02-3.47248E+00-1.33975E+02 4.95266E-01                                    PTN 2 22
 1.32694E+02 4.95266E-01-1.32694E+02-3.44402E+00                                    PTN 2 22
C       3RD K, 2ND M
C       AIC PARTITION CONTROL
    6    1
C       AIC PARTITION CONTROL
    3    0
C       AIC PARTITION
 2.19199E+01-2.50276E+00-3.02671E+01 2.21519E-01 8.34713E+00 1.92716E-02PTN 3 12
 6.69688E+00 2.21519E-01 6.24291E+00-3.19634E+00-1.29398E+01-2.98280E-01PTN 3 13
-1.83468E+00 1.92716E-02 1.04602E+01-2.98280E-01-8.62551E+00-8.52633E-01PTN 3 14
C       AIC PARTITION CONTROL
    1    1
C       AIC PARTITION CONTROL
    2    0
C       AIC PARTITION
 8.37342E+00-8.68120E-01-8.37342E+00 1.23816E-01                                    PTN 3 12
 8.29337E+00 1.23816E-01-8.29337E+00-8.61005E-01                                    PTN 3 12
C       AIC PARTITION CONTROL
    2    0
C       AIC PARTITION
 8.37342E+00-8.68120E-01-8.37342E+00 1.23816E-01                                    PTN 3 22
 8.29337E+00 1.23816E-01-8.29337E+00-8.61005E-01                                    PTN 3 22
```

```
C       4TH K, 2ND M
C       AIC PARTITION CONTROL
    6     1
C       AIC PARTITION CONTROL
    3     0
C       AIC PARTITION
 3.50719E+00-1.00111E+00-4.84273E+00 8.86074E-02 1.33554E+00 7.70863E-03PTN 4 12
 1.07150E+00 8.86074E-02 9.98866E-01-1.27854E+00-2.07037E+00-1.19312E-01PTN 4 13
-2.93549E-01 7.70863E-03 1.67363E+00-1.19312E-01-1.38008E+00-3.41053E-01PTN 4 14
C       AIC PARTITION CONTROL
    1     1
C       AIC PARTITION CONTROL
    2     0
C       AIC PARTITION
 1.33975E+00-3.47248E-01-1.33975E+00 4.95266E-02                        PTN 4 12
 1.32694E+00 4.95266E-02-1.32694E+00-3.44402E-01                        PTN 4 12
C       AIC PARTITION CONTROL
    2     0
C       AIC PARTITION
 1.33975E+00-3.47248E-01-1.33975E+00 4.95266E-02                        PTN 4 22
 1.32694E+00 4.95266E-02-1.32694E+00-3.44402E-01                        PTN 4 22
C       1ST K, 3RD M
C       AIC PARTITION CONTROL
    6     1
C       AIC PARTITION CONTROL
    3     0
C       AIC PARTITION
 6.58877E+05-3.76143E+02-9.09727E+05 3.33141E+01 2.50850E+05 2.85050E+00PTN 1 12
 2.00614E+05 3.33141E+01 1.87349E+05-4.79253E+02-3.87963E+05-4.46293E+01PTN 1 13
-5.48687E+04 2.85050E+00 3.13149E+05-4.46293E+01-2.58281E+05-1.27688E+02PTN 1 14
C       AIC PARTITION CONTROL
    1     1
C       AIC PARTITION CONTROL
    2     0
C       AIC PARTITION
 2.51605E+05-1.30415E+02-2.51605E+05 1.85907E+01                        PTN 1 12
 2.48402E+05 1.85907E+01-2.48402E+05-1.28992E+02                        PTN 1 12
C       AIC PARTITION CONTROL
    2     0
C       AIC PARTITION
 2.51605E+05-1.30415E+02-2.51605E+05 1.85907E+01                        PTN 1 22
 2.48402E+05 1.85907E+01-2.48402E+05-1.28992E+02                        PTN 1 22
C       2ND K, 3RD M
C       AIC PARTITION CONTROL
    6     1
C       AIC PARTITION CONTROL
    3     0
C       AIC PARTITION
 2.63551E+02-7.52286E+00-3.63891E+02 6.66282E-01 1.00340E+02 5.70099E-02PTN 2 12
 8.02458E+01 6.66282E-01 7.49395E+01-9.58506E+00-1.55185E+02-8.92587E-01PTN 2 13
-2.19475E+01 5.70099E-02 1.25260E+02-8.92587E-01-1.03312E+02-2.55375E+00PTN 2 14
C       AIC PARTITION CONTROL
    1     1
C       AIC PARTITION CONTROL
    2     0
C       AIC PARTITION
 1.00642E+02-2.60830E+00-1.00642E+02 3.71814E-01                        PTN 2 12
 9.93608E+01 3.71814E-01-9.93608E+01-2.57983E+00                        PTN 2 12
C       AIC PARTITION CONTROL
    2     0
C       AIC PARTITION
 1.00642E+02-2.60830E+00-1.00642E+02 3.71814E-01                        PTN 2 22
 9.93608E+01 3.71814E-01-9.93608E+01-2.57983E+00                        PTN 2 22
C       3RD K, 3RD M
C       AIC PARTITION CONTROL
    6     1
```

```
C     AIC PARTITION CONTROL
   3     0
C     AIC PARTITION
 1.64719E+01-1.88072E+00-2.27432E+01 1.66571E-01 6.27126F+00 1.42525E-02PTN 3 12
 5.01536E+00 1.66571E-01 4.68372E+00-2.39627E+00-9.69908E+00-2.23147E-01PTN 3 13
-1.37172E+00 1.42525E-02 7.82873E+00-2.23149E-01-6.45701E+00-6.38438E-01PTN 3 14
C     AIC PARTITION CONTROL
   1     1
C     AIC PARTITION CONTROL
   2     0
C     AIC PARTITION
 6.29012E+00-6.52075E-01-6.29012E+00 9.29536E-02                         PTN 3 12
 6.21005E+00 9.29536E-02-6.21005E+00-6.44958E-01                         PTN 3 12
C     AIC PARTITION CONTROL
   2     0
C     AIC PARTITION
 6.29012E+00-6.52075E-01-6.29012E+00 9.29536E-02                         PTN 3 22
 6.21005E+00 9.29536E-02-6.21005E+00-6.44958E-01                         PTN 3 22
C     4TH K, 3RD M
C     AIC PARTITION CONTROL
   6     1
C     AIC PARTITION CONTROL
   3     0
C     AIC PARTITION
 2.63551E+00-7.52286E-01-3.63891E+00 6.66282E-02 1.00340E+00 5.70099E-03PTN 4 12
 8.02458E-01 6.66282E-02 7.49395E-01-9.58506E-01-1.55185E+00-8.92587E-02PTN 4 13
-2.19475E-01 5.70099E-03 1.25260E+00-8.92587E-02-1.03312E+00-2.55375E-01PTN 4 14
C     AIC PARTITION CONTROL
   1     1
C     AIC PARTITION CONTROL
   2     0
C     AIC PARTITION
 1.00642E+00-2.60830E-01-1.00642E+00 3.71814E-02                         PTN 4 12
 9.93608E-01 3.71814E-02-9.93608E-01-2.57983E-01                         PTN 4 12
C     AIC PARTITION CONTROL
   2     0
C     AIC PARTITION
 1.00642E+00-2.60830E-01-1.00642E+00 3.71814E-02                         PTN 4 22
 9.93608E-01 3.71814E-02-9.93608E-01-2.57983E-01                         PTN 4 22
C     MACH NUMBERS FOR VELOCITIES AND EIGENVECTOR REQUEST
 2.5           -3.0           3.5
```

Table 8 - Output from Program MPASES for Aero-Servo-Elastic Stability Analysis

```
TYPICAL AIR-TO-AIR MISSILE

AERO-SERVO-ELASTIC ANALYSIS USING PISTON THEORY

                    AERO-SERVO-ELASTIC STABILITY ANALYSIS

                          14 DEGREES OF FREEDOM
                           5 FLEXIBLE MODES
                           2 RIGID BODY MODES
                           1 CONTROL SURFACES
                          11 SERVO ELEMENTS

                    SHIFT EIGENVALUE(GAMMA) = -1.000E+02

                          30 MODES REQUESTED


                          1 ALTITUDE VARIATIONS

              AIC MATRICES FOR 3 MACH NUMBERS AND  4 REDUCED FREQUENCIES

                 REFERENCE SEMI-CHORD = 1.50000E+00 FT
                            SEMI-SPAN = 1.00000E+00 FT
                         SURFACE AREA = 3.00000E+00 SQ FT

                 MACH NUMBER DEVIATION = 5.00000E-03

UPPER TRIANGLE OF WEIGHT MATRIX

ROW  1
 5.00000E+01  0.   0.   0.   0.   0.   0.   0.
 0.           0.   0.   0.   0.   0.
ROW  2
 5.00000E+01  0.   0.   0.   0.   0.   0.   0.
 0.           0.   0.   0.   0.
ROW  3
 5.00000E+01  0.   0.   0.   0.   0.   0.   0.
 0.           0.   0.   0.
ROW  4
 5.00000E+01  0.   0.   0.   0.   0.   0.   0.
 0.           0.   0.
ROW  5
 5.00000E+01  0.   0.   0.   0.   0.   0.   0.
 0.           0.
ROW  6
 5.00000E+01  0.   0.   0.   0.   0.   0.   0.
 0.
```

AIC MATRIX FOR MACH NUMBER = 2.0000E+00 AND REDUCED FREQUENCY = 1.000000E-03

0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	1.31264E+06	-7.49339E+02	-1.81260E+06	6.62394E+01
4.99957E+05	5.88198E+00	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	4.02395E+05	6.62394E+01	3.74455E+05	-9.59263E+02
-7.76849E+05	-8.97240E+01	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	-1.10424E+05	5.88198E+00	6.28925E+05	-8.97240E+01
-5.18501E+05	-2.56280E+02	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	5.01602E+05	-2.60043E+02	-5.01602E+05	3.71087E+01
0.	0.	0.	0.				

0.	0.	0.	0.				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	4.98401E+05	3.71087E+01	-4.98401E+05	-2.58620E+02
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
5.01602E+05	-2.60063E+02	-5.01602E+05	3.71087E+01				
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
0.	0.	0.	0.	0.	0.	0.	0.
4.98401E+05	3.71087E+01	-4.98401E+05	-2.58620E+02				

STABILITY ANALYSIS RESULTS

SPEED OF SOUND = 1.10000E+03 FT/SEC

DENSITY = 2.37800E-03 SLUGS/CU FT

MACH NUMBER = 2.50000E+00

VELOCITY = 2.75000E+03 FT/SEC, 1.62824E+03 KNOTS

MODE	EIGENVALUE-R MU-RPS	EIGENVALUE-I OMEGA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	REDUCED FREQUENCY	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-2.610250E-09	0.	0.	6.154342E-10	0.		2.655462E+08
2	-4.602061E-05	0.	0.	7.325394E-06	0.		1.505966E+04
3	5.623010E+00	0.	0.	8.950254E-01	0.		-1.232566E-01
4	-8.273252E+01	0.	0.	1.316729E+01	0.		8.378171E-03
5	-2.915276E+02	0.	0.	4.639806E+01	0.		2.377638E-03
6	-3.331353E+02	0.	0.	5.302013E+01	0.		2.080670E-03
7	-9.985408E+02	0.	0.	1.589227E+02	0.		6.941601E-04
8	5.138772E+16	0.	0.	8.178609E+15	0.		-1.348858E-17
9	-7.812532E+16	0.	0.	1.243403E+16	0.		8.872248E-18
10	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
11	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
12	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
13	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
14	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
15	-1.300318E+01	3.989982E+01	6.350253E+00	6.678964E+00	2.176353E-02	3.098563E-01	5.330547E-02
16	-1.379457E+01	2.498730E+02	3.976862E+01	3.982918E+01	1.362947E-01	5.512303E-02	5.024639E-02
17	-2.454961E+02	3.240143E+02	5.156848E+01	6.469813E+01	1.767351E-01	6.038964E-01	2.823524E-03
18	-4.424164E+02	4.318050E+02	6.872390E+01	9.839172E+01	2.355380E-01	7.156370E-01	1.566730E-03
19	-2.152869E+01	6.258353E+02	9.960478E+01	9.966369E+01	3.413647E-01	3.437863E-02	3.219734E-02
20	-8.811391E+00	7.552225E+02	1.202453E+02	1.202531E+02	4.320668E-01	1.110766E-02	7.866490E-02
21	-9.890447E+01	9.602461E+02	1.528270E+02	1.536356E+02	5.237673E-01	1.024633E-01	7.007860E-03
22	-2.423488E+01	1.562329E+03	2.486523E+02	2.486622E+02	8.521792E-01	1.551816E-02	2.860123E-02

STABILITY ANALYSIS RESULTS

SPEED OF SOUND = 1.10000E+03 FT/SEC

DENSITY = 2.37000E-03 SLUGS/CU FT

MACH NUMBER = 3.00000E+00

VELOCITY = 3.30000E+03 FT/SEC, 1.95300E+03 KNOTS

MODE	EIGENVALUE-R MU-RPS	EIGENVALUE-I OMEGA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	REDUCED FREQUENCY	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-1.333542E-07	0.	0.	2.122390E-08	0.		5.197790E+06
2	-2.833307E-04	0.	0.	4.509443E-05	0.		2.446373E+03
3	6.273310E+00	0.	0.	9.984282E-01	0.		-1.104915E-01
4	-8.495974E+01	0.	0.	1.352170E+01	0.		8.158537E-03
5	-2.903340E+02	0.	0.	4.620822E+01	0.		2.387408E-03
6	-3.339183E+02	0.	0.	5.314474E+01	0.		2.075799E-03
7	-9.985341E+02	0.	0.	1.589216E+02	0.		6.941048E-04
8	-2.244247E+16	0.	0.	3.571831E+15	0.		3.088551E-17
9	1.419041E+17	0.	0.	2.258473E+16	0.		-4.884618E-18
10	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
11	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
12	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
13	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
14	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
15	-1.409177E+01	4.235111E+01	6.740388E+00	7.103723E+00	1.925049E-02	3.157183E-01	4.918808E-02
16	-1.383541E+01	2.467080E+02	3.926479E+01	3.932644E+01	1.121408E-01	5.599213E-02	5.009950E-02
17	-2.433659E+02	3.278806E+02	5.218478E+01	6.498837E+01	1.490394E-01	5.959973E-01	2.848168E-03
18	-4.428526E+02	4.305961E+02	6.853150E+01	9.830720E+01	1.957255E-01	7.169585E-01	1.565187E-03
19	-2.005927E+01	6.258694E+02	9.960980E+01	9.966095E+01	2.844849E-01	3.203393E-02	3.455495E-02
20	-8.833533E+00	7.931907E+02	1.262402E+02	1.262481E+02	3.605412E-01	1.113602E-02	7.846771E-02
21	-9.704257E+01	9.937763E+02	1.581644E+02	1.589167E+02	4.517165E-01	9.718804E-02	7.142713E-03
22	-2.421916E+01	1.562332E+03	2.486529E+02	2.486820E+02	7.101511E-01	1.550006E-02	2.861970E-02

EIGENVECTORS FOR MODES REQUESTED

EIGENVECTOR FOR MODE 3

-4.6300327E-05	0.	7.6834054E-06	0.	1.5546321E-06	0.	-5.9769240E-04	0.
-4.0300253E-04	0.	2.8967084E-02	0.	-5.6470703E-05	0.	-1.8337755E-04	0.
-3.3939563E-04	0.	-3.6597727E-05	0.	1.0000000E+00	0.	-7.3805205E-06	0.
1.2247770E-06	0.	2.4781084E-07	0.	-9.5275465E-05	0.	-6.4253570E-05	0.
4.6175123E-03	0.	-9.8817402E-06	0.	-2.9231309E-05	0.	-5.4101520E-05	0.
-5.8338787E-06	0.	1.5940549E-01	0.	-5.3764248E-05	0.	-5.3764248E-06	0.
-3.7999308E-05	0.	-3.2165430E-05	0.	-1.0752950E-04	0.	-2.3830142E-04	0.
-1.5940549E-01	0.	-3.4590492E-04	0.				

EIGENVECTOR FOR MODE 4

-6.8504623E-05	0.	7.1592704E-04	0.	1.4431766E-06	0.	-1.2348734E-03	0.
-8.8802044E-04	0.	5.4677128E-04	0.	-1.0940390E-05	0.	-2.8403144E-04	0.
1.6506617E-03	0.	-4.9784788E-05	0.	1.0000000E+00	0.	4.0631889E-07	0.
-8.4266632E-04	0.	-1.0966595E-08	0.	1.4534813E-05	0.	1.0216047E-05	0.
-6.4591919E-06	0.	1.2877156E-07	0.	3.3525488E-06	0.	-1.2366584E-05	0.
5.8598097E-07	0.	-1.1770281E-02	0.	-1.3514797E-05	0.	-1.3514797E-06	0.
-6.1677671E-07	0.	-1.2047577E-04	0.	-2.7029595E-05	0.	5.2571106E-05	0.
1.1770281E-02	0.	2.5541511E-05	0.				

EIGENVECTOR FOR MODE 5

-2.3125109E-07	0.	3.8076111E-08	0.	8.8367507E-09	0.	-7.8383109E-06	0.
-7.3525011E-06	0.	3.1184235E-07	0.	-6.4884470E-09	0.	-2.1740099E-06	0.
4.9143183E-07	0.	-4.4014198E-07	0.	1.0000000E+00	0.	7.9449799E-10	0.
-1.3114553E-10	0.	-3.8436443E-11	0.	2.8497490E-08	0.	2.5324215E-08	0.
-1.8742596E-09	0.	2.2320772E-11	0.	7.4879411E-09	0.	-1.4926384E-09	0.
1.5366466E-09	0.	-3.4442494E-03	0.	-2.3851224E-09	0.	-2.3851224E-10	0.
-2.5759571E-08	0.	-2.1296218E-08	0.	-4.7702448E-09	0.	7.4789000E-06	0.
3.4442994E-03	0.	7.4741649E-04	0.				

EIGENVECTOR FOR MODE 6

-1.2974400E-07	0.	2.4247764E-08	0.	5.9078842E-09	0.	-5.0733309E-06	0.
-5.2427457E-06	0.	1.5342704E-07	0.	-3.1622657E-09	0.	-1.5154206E-06	0.
6.7434188E-08	0.	-8.1882165E-08	0.	1.0000000E+00	0.	3.8055019E-10	0.
-7.2615883E-11	0.	-1.7692666E-11	0.	1.5193332E-08	0.	1.5700666E-08	0.
-4.5947484E-10	0.	9.5300742E-12	0.	4.5382983E-09	0.	-2.0194820E-10	0.
2.4497662E-10	0.	-2.9947450E-03	0.	-3.0318830E-10	0.	-3.0318830E-11	0.
-1.9463455E-08	0.	-1.9708432E-08	0.	-6.0637661E-10	0.	6.4992024E-06	0.
2.9947450E-03	0.	6.4985966E-06	0.				

EIGENVECTOR FOR MODE 7

2.7067257E-04	0.	-2.2547111E-04	0.	-1.4632794E-06	0.	4.4608503E-02	0.
3.2587933E-01	0.	-1.3862653E-04	0.	2.1733386E-06	0.	6.0886652E-02	0.
-1.2684000E-03	0.	1.0000000E+00	0.	9.5817840E-01	0.	-2.7106994E-07	0.
2.2580212E-07	0.	1.4654276E-07	0.	-4.4673991E-05	0.	-3.2635774E-06	0.
1.3883005E-07	0.	-2.1765292E-09	0.	-6.0484847E-05	0.	1.2702621E-06	0.
-1.0014681E-03	0.	-9.5958507E-04	0.	8.6652781E-04	0.	8.6652781E-05	0.
1.7335165E-06	0.	1.0032016E-03	0.	1.7338556E-03	0.	-1.7309733E-03	0.
4.5958507E-04	0.	2.0622996E-06	0.				

EIGENVECTOR FOR MODE 15

8.3924602E-06	6.0051975E-05	2.9894313E-06	-4.0584939E-06	6.1135582E-07	-7.9655900E-07	3.6419061E-04	1.2263898E-03
2.6026056E-04	6.4994025E-04	2.2083859E-03	1.1697641E-03	-5.8057831E-05	-2.2321618E-05	2.7364978E-05	2.4347742E-04
1.5027427E-03	-2.5390097E-03	2.9249475E-04	-1.3490084E-04	1.0000000E+00	0.	1.3073204E-06	-6.5977641E-07
-1.0027647E-07	-3.4560724E-08	-2.1258849E-08	-7.3620060E-09	2.3452018E-05	-1.0402806E-05	1.6227338E-05	-1.1545353E-05
9.6710251E-06	-5.5362807E-05	-5.9614551E-06	1.4048530E-06	4.9824041E-06	-2.3039746E-06	-6.5105849E-05	-1.5683144E-05
-4.9360794E-06	-5.2437435E-06	-7.0734921E-03	-2.1258522E-02	-6.6677847E-05	-1.3043064E-05	-6.6677847E-06	-1.3043064E-06
4.8336435E-07	-3.0720793E-06	5.4202438E-06	1.5916641E-06	-1.3335569E-04	-2.6086127E-05	1.4870517E-04	7.2217120E-05
7.0735921E-03	2.1258522E-02	1.5349478E-05	4.6130992E-05				

EIGENVECTOR FOR MODE 16

2.7591134E-05	7.0659749E-05	-1.2254122E-06	-1.7520450E-06	-2.1461577E-07	-3.0815515E-07	2.2932317E-04	2.9490970E-04
1.1053660E-04	1.4369140E-04	1.3593380E-05	1.4445010E-05	-2.6312050E-07	-2.9690547E-07	3.0294694E-05	5.2362531E-06
1.4490163E-04	-6.1970050E-04	1.0210441E-04	-1.0347994E-04	1.0000000E+00	0.	3.0350437E-07	-1.2005779E-07
-9.0699307E-09	5.3490800E-09	-1.1905203E-09	9.3701927E-10	1.1394099E-06	-9.9345916E-07	5.5556322E-07	-4.7920261E-07
5.5209054E-08	-5.0199756E-08	-1.1355429E-09	1.2112754E-09	2.0205952E-07	-1.0050746E-07	-2.5371861E-06	-4.4505501E-07
-7.0266470E-07	-6.9456990E-07	-2.2660109E-04	-4.0400670E-03	-2.5275001E-06	1.0100365E-07	-2.5275001E-07	1.0100365E-08
3.2709933E-08	-2.4317333E-06	8.1537463E-07	9.7025250E-07	-5.0550001E-06	3.0200730E-07	5.5467245E-06	8.5002914E-06
2.2660109E-04	4.0400670E-03	4.9172437E-07	8.7082487E-06				

EIGENVECTOR FOR MODE 17

1.1266374E-05	1.0456035E-05	-3.4060456E-06	6.0202507E-07	-5.4065014E-07	3.2426518E-07	6.7340056E-04	-2.3093227E-04
2.1027571E-04	-3.7203349E-04	-1.1995518E-06	-1.3806244E-05	2.3001593E-08	2.4590213E-07	8.7206202E-05	-1.0570290E-04
-5.0424309E-04	-1.5732315E-04	-3.0090347E-04	-7.0100819E-04	1.0000000E+00	0.	1.5910095E-08	-4.6175375E-08
6.3137720E-09	5.7040703E-09	1.4355460E-09	6.0109240E-10	-1.4527429E-06	-9.7549690E-07	-1.0390950E-06	1.3030540E-07
-2.5509304E-08	2.2429950E-08	5.4022950E-10	-4.7724420E-10	-3.3526560E-07	-1.7365707E-08	9.2601040E-07	1.2212185E-06
-1.0509706E-06	1.7037056E-06	-1.4595029E-03	-1.9604943E-03	1.0035355E-06	1.1531327E-06	1.0035355E-07	1.1531327E-07
2.3300560E-09	-4.9070764E-09	1.0013140E-06	-1.7000134E-06	2.1270711E-06	2.3062653E-06	1.0402239E-06	1.9610273E-06
1.4595029E-03	1.9604943E-03	3.1672950E-06	4.2672927E-06				

EIGENVECTOR FOR MODE 18

6.2300740E-06	1.7253100E-05	-6.0071620E-06	-1.3512493E-06	-1.2815459E-06	7.4923219E-07	1.5198775E-03	9.3308234E-05
3.0131672E-04	-1.0340021E-03	-9.4321275E-07	-1.3631508E-05	9.9530696E-09	2.9276235E-07	2.3154834E-04	-3.9364192E-04
-1.1233797E-04	6.1139045E-05	-1.3012614E-03	-3.9684594E-03	1.0000000E+00	0.	1.2239018E-08	-2.7058092E-08
5.5494450E-09	8.5303939E-09	2.3331035E-09	5.7670013E-10	-1.0568503E-06	-1.0236376E-06	-2.2647364E-06	1.5662712E-06
-1.4280490E-08	1.6080914E-08	3.1565059E-10	-3.5104422E-10	-7.1308403E-07	1.9564227E-07	1.9939484E-07	5.5814980E-08
-2.9909671E-06	6.6981019E-06	-1.1607237E-03	-1.1285992E-03	2.7253046E-07	-1.1044041E-07	2.7253046E-08	-1.1044041E-08
7.2236190E-10	-5.3265906E-09	2.9916894E-06	-6.1034205E-06	5.4500692E-07	-2.2084082E-07	1.9737095E-06	2.6699411E-06
1.1607237E-03	1.1285992E-03	2.5187784E-06	2.4490082E-06				

EIGENVECTOR FOR MODE 19

-5.8101379E-06	9.5676055E-06	-5.1954773E-06	6.0195639E-06	-2.9049591E-07	4.6021249E-07	5.2300054E-04	-8.1787257E-04
-4.0204104E-05	-2.0171333E-05	3.5580727E-06	-5.0191081E-06	-7.7569059E-08	1.2183707E-07	3.2019024E-06	-2.3020714E-06
-4.6067205E-06	-5.0769205E-05	-2.3002504E-05	-2.7134754E-05	1.0000000E+00	0.	1.5560637E-08	8.7843646E-09
1.0031409E-08	7.9540977E-09	7.0219597E-10	4.3972104E-10	-1.3322308E-06	-7.9422135E-07	-4.2904700E-08	6.5740409E-08
-9.1505191E-09	-5.3750710E-09	1.9463407E-10	1.1757951E-10	-3.0345151E-09	-4.9920904E-09	-9.0372217E-08	1.0304044E-08
-4.2031932E-08	4.1295747E-08	-5.1157040E-05	-1.5961444E-03	-6.0695492E-08	4.9362311E-08	-6.0695492E-09	4.9362311E-09
5.3569912E-09	-5.4308554E-10	4.7340923E-08	-4.1838032E-08	-1.2139098E-07	9.8724621E-08	2.3240178E-07	3.3649887E-06
5.1157040E-05	1.5961444E-03	1.1101078E-07	3.4636333E-06				

EIGENVECTOR FOR MODE 20

1.1204204E-07	-6.0960057E-08	-8.2135241E-06	7.7402784E-06	5.0004601E-09	4.5455103E-10	-2.4049801E-05	2.1210400E-05
-9.1099057E-07	4.1042054E-04	-7.1303144E-04	5.4615172E-08	1.5057519E-09	-1.3030805E-09	4.3067654E-07	-3.9144980E-07
-5.0413920E-07	-1.0742281E-05	-5.2902473E-06	-2.5099872E-06	1.0000000E+00	0.	-8.0502217E-11	-1.4027024E-10
9.0825093E-09	1.0244983E-08	4.9156479E-13	-7.3162931E-12	2.7074916E-08	3.0016900E-08	5.2059633E-09	9.6357250E-10
6.9640332E-11	0.9191032E-11	-1.0442730E-12	-1.0794133E-12	-5.9961005E-10	-5.4748907E-10	-1.3534347E-08	7.0531211E-10
-3.0090955E-09	6.7140720E-09	-1.4030649E-05	-1.2605745E-03	-7.9370092E-09	7.1450530E-09	-7.9370092E-10	7.1450530E-10
3.4295690E-09	-9.6613919E-11	6.5192153E-09	-6.8100059E-09	-1.5874170E-08	1.4290100E-08	4.0330047E-08	2.7211565E-06
1.5030649E-05	1.2605745E-03	3.0403069E-08	2.7354400E-06				

EIGENVECTOR FOR MODE 21

3.7074011E-06	-2.0921000E-06	-7.0057900E-06	6.0600019E-06	3.3376000E-06	-1.5669090E-07	-2.4230609E-03	4.0450107E-06
-2.0020060E-03	2.5060649E-04	-1.9439028E-06	1.6491602E-06	3.5006967E-08	-3.4073041E-08	-2.9670409E-07	1.0702279E-06
-1.5394130E-05	3.4080706E-05	1.7110175E-05	7.5045242E-06	1.0000000E+00	0.	-3.0520410E-09	-3.5131040E-09
7.0003594E-09	7.1119114E-09	-4.0100055E-10	-3.3110150E-09	7.1000020E-07	2.3000450E-06	5.3054340E-07	2.7600204E-06
1.0330000E-09	1.7771434E-09	-3.7370906E-11	-3.1570924E-11	1.1016243E-09	1.9105260E-10	3.6265900E-08	1.1949152E-08
5.0147472E-09	-1.7705146E-06	-9.7333722E-05	-9.9675746E-04	2.4769405E-08	-1.4005423E-08	2.4749405E-09	-1.4005423E-09
2.1001320E-09	-3.7444073E-10	-3.0400051E-09	1.7410700E-08	4.9450970E-08	-2.0010040E-08	1.0171521E-07	2.1909750E-06
9.7333722E-05	9.9675794E-04	2.1121410E-07	2.1029040E-06				

EIGENVECTOR FOR MODE 22

2.9759990E-08	-1.3145316E-08	-2.1100031E-09	1.6504250E-08	1.5007557E-05	-6.3155032E-06	9.6095004E-06	-3.1627033E-06
1.6405254E-05	2.2200000E-06	-3.2760054E-06	8.6619190E-09	8.7054000E-10	-1.9250923E-10	4.0410043E-07	7.6905509E-07
5.0457789E-06	9.9718145E-05	1.0910097E-05	-1.1122479E-05	1.0000000E+00	0.	-8.7070002E-12	-1.0967045E-11
1.0342250E-11	1.1000055E-12	-4.1900531E-09	-4.5793044E-09	-2.1070344E-09	-4.1564049E-09	1.2614000E-09	-1.0571250E-08
5.0000066E-12	2.0090521E-11	-1.3100470E-13	-5.5900519E-13	4.0732747E-10	-3.1741714E-10	6.3755120E-08	-4.6020133E-09
-7.3050930E-09	-1.1996049E-08	-9.9199136E-06	-6.3991400E-06	1.6215930E-08	-3.0679723E-08	1.6215930E-09	-3.0679723E-09
9.2790025E-10	-7.5092394E-12	8.2330073E-09	1.1907404E-08	3.2931059E-08	-6.1359947E-08	-1.6905657E-08	1.5597247E-06
9.9199136E-06	6.3991400E-04	2.1526213E-08	1.3000153E-06				

STRUCTURAL DEFLECTIONS AT SYSTEM MASS POINTS FOR EACH MODE

DEFLECTIONS FOR MODE 3

1.0000000E+00	0.	9.7100762E-01	0.	9.4197134E-01	0.	9.1204902E-01	0.
8.0332200E-01	0.	8.5410034E-01	0.	8.2423077E-01	0.	7.9307450E-01	0.
7.6300592E-01	0.	7.3200269E-01	0.	7.2054773E-01	0.	7.4650471E-01	0.
7.1401043E-01	0.	7.3252541E-01	0.				

DEFLECTIONS FOR MODE 4

-1.6409973E-01	0.	-1.1135603E-01	0.	-5.7573072E-02	0.	-1.6025344E-03	0.
5.7757192E-02	0.	1.2194440E-01	0.	1.9251179E-01	0.	2.7024493E-01	0.
3.5412471E-01	0.	4.4131601E-01	0.	6.6578331E-01	0.	2.1037735E-01	0.
1.0000000E+00	0.	5.6459404E-01	0.				

DEFLECTIONS FOR MODE 5

-6.2607020E-03	0.	-7.0480360E-03	0.	-7.0077740E-03	0.	-5.7679900E-03	0.
-2.4030031E-03	0.	4.0035400E-03	0.	1.5715781E-02	0.	3.4309792E-02	0.
5.9414151E-02	0.	8.2251356E-02	0.	4.8030736E-01	0.	-3.5600560E-01	0.
1.0000000E+00	0.	1.0436300E-01	0.				

DEFLECTIONS FOR MODE 6

-2.9930564E-03	0.	-4.4550349E-03	0.	-5.2363375E-03	0.	-4.9600070E-03	0.
-3.2450540E-03	0.	9.9715770E-04	0.	9.7796061E-03	0.	2.4097473E-02	0.
4.0199279E-02	0.	7.1079713E-02	0.	4.7590090E-01	0.	-4.1395993E-01	0.
1.0000006E+00	0.	1.1005117E-01	0.				

DEFLECTIONS FOR MODE 7

-3.9771040E-04	0.	3.7230153E-04	0.	2.9900502E-04	0.	-3.7492500E-04	0.
-3.5047195E-04	0.	1.1494071E-03	0.	2.9710063E-03	0.	2.6764354E-03	0.
-1.1045426E-03	0.	-7.5302775E-03	0.	-3.7045396E-01	0.	1.0000000E+00	0.
-6.6302320E-01	0.	7.1403060E-01	0.				

DEFLECTIONS FOR MODE 15

-3.1132647E-01	-5.7734360E-02	-1.6532390E-01	-5.3160250E-02	-1.9621942E-02	-4.8962950E-02	1.2539447E-01	-4.5577950E-02
2.6937093E-01	-4.3448231E-02	4.1216739E-01	-4.3023187E-02	5.5387467E-01	-4.4700613E-02	6.9481126E-01	-4.0598784E-02
8.3533066E-01	-5.4322168E-02	9.7571937E-01	-6.1003579E-02	8.8808519E-01	-1.3304455E-01	1.0000000E+00	0.
8.0939736E-01	-2.3567278E-01	9.2131217E-01	-1.0262823E-01				

DEFLECTIONS FOR MODE 16

2.0640092E-01	3.4913986E-02	9.4867305E-02	1.5254147E-02	-8.0843179E-03	-6.8848393E-03	-9.1731797E-02	-2.3384697E-02
-1.4587023E-01	-3.1372645E-02	-1.0323473E-01	-2.9160401E-02	-1.4128796E-01	-1.6789923E-02	-8.3725040E-02	4.0363057E-03
-4.1498067E-04	3.0449221E-02	9.5247605E-02	5.9349825E-02	4.5120589E-01	3.2730714E-02	-1.6151555E-01	5.7325790E-02
1.0000000E+00	0.	3.8727856E-01	2.4595077E-02				

DEFLECTIONS FOR MODE 17

6.3903009E-03	9.0710005E-03	4.9636948E-03	1.2612316E-03	3.0706444E-03	-5.6336665E-03	3.6908314E-05	-1.0179152E-02
-4.4425267E-03	-1.0682007E-02	-9.4273692E-03	-5.8384267E-03	-1.2638160E-02	4.6701495E-03	-1.1741821E-02	1.9987166E-02
-6.1748290E-03	3.8462225E-02	2.2630860E-03	5.8315176E-02	4.0913681E-01	3.2570014E-03	-2.7110221E-01	2.9852942E-01
1.0000000E+00	0.	3.1976098E-01	2.9527281E-01				

DEFLECTIONS FOR MODE 18

9.3688341E-05	2.1964529E-03	1.0814025E-03	4.5720362E-04	1.7173941E-03	-1.5189846E-03	1.3428658E-03	-3.3753729E-03
-5.4048996E-04	-4.0267895E-03	-3.4196362E-03	-2.2803306E-03	-5.4491502E-03	2.3398821E-03	-4.5409442E-03	9.3205953E-03
1.5953439E-05	1.7633078E-02	6.8971505E-03	2.6404125E-02	4.2696692E-01	-5.5859306E-02	-3.8383536E-01	5.9422020E-01
1.0000000E+00	0.	1.4919771E-01	6.5007951E-01				

DEFLECTIONS FOR MODE 19

-1.4151377E-02	-4.2278339E-04	-3.8640071E-04	2.1703162E-04	1.0103660E-02	5.7973785E-04	1.3837397E-02	4.8206719E-04
4.7965932E-03	-9.8803656E-06	2.0636647E-04	-5.8261704E-04	-1.0265063E-02	-8.4863983E-04	-1.7128046E-02	-5.6473934E-04
-1.8762145E-02	2.4674273E-04	-1.6992755E-02	1.3485235E-03	3.2470742E-01	8.8051821E-03	2.9698624E-01	-6.3820254E-02
1.0000000E+00	0.	9.7227082E-01	-7.2025437E-02				

DEFLECTIONS FOR MODE 20

4.5543903E-01	2.4866308E-02	-6.4447191E-02	-4.0222795E-03	-4.1562572E-01	-2.2302309E-02	-4.5089466E-01	-2.3662014E-02
-1.0948871E-01	-9.1314103E-03	1.4271415E-01	1.1512050E-02	4.5149714E-01	2.5846422E-02	4.1172981E-01	2.2239323E-02
6.0149276E-02	2.6322596E-03	-4.7638303E-01	-2.6533356E-02	7.1170501E-02	3.6790782E-02	1.3592084E-01	-7.5898154E-02
9.3524466E-01	1.1268894E-01	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 21

1.7266192E-03	-8.5428916E-04	-7.3827592E-04	1.5922186E-04	-1.9101191E-03	8.3197612E-04	-1.2663339E-03	8.0490111E-04
4.0519612E-04	3.1417415E-04	1.0749458E-03	-5.1606352E-04	1.6625956E-03	-1.0261526E-03	5.3269319E-04	-7.4102638E-04
-1.0261898E-03	4.8824908E-04	-2.5811668E-03	2.0090129E-03	4.7366386E-02	-1.2962172E-02	6.8737995E-01	1.6742704E-02
3.5598644E-01	-3.1704876E-02	1.0000000E+00	0.				

DEFLECTIONS FOR MODE 22

-8.0970621E-01	1.9312853E-03	-6.9302457E-01	-1.2583487E-03	9.9688223E-01	-2.3338632E-03	1.0585984E-01	-7.9123974E-04
-9.4445934E-01	1.5751016E-03	-9.4407476E-01	2.4568826E-03	1.0728119E-01	1.3696919E-03	1.0000000E+00	0.
6.4652846E-01	3.1881978E-05	-7.9242021E-01	1.4884990E-03	-2.0794086E-01	-1.2081892E-02	-1.5489633E-01	1.6678605E-01
6.9422824E-01	2.1460754E-02	2.5726397E-01	2.0025970E-01				

STABILITY ANALYSIS RESULTS

SPEED OF SOUND = 1.10000E+03 FT/SEC

DENSITY = 2.37800E-03 SLUGS/CU FT

MACH NUMBER = 3.50000E+00

VELOCITY = 3.85000E+03 FT/SEC, 2.27953E+03 KNOTS

MODE	EIGENVALUE-R NU-RPS	EIGENVALUE-I OMEGA-RPS	DAMPED FREQUENCY-CPS	UNDAMPED FREQUENCY-CPS	REDUCED FREQUENCY	DAMPING RATIO ZETA	TIME TO 1/2 AMPLITUDE
1	-1.005747E-07	0.	0.	1.600695E-08	0.		6.891868E+06
2	-2.810003E-04	0.	0.	4.472259E-05	0.		2.466713E+03
3	6.923673E+00	0.	0.	1.101937E+00	0.		-1.001126E-01
4	-8.715872E+01	0.	0.	1.387047E+01	0.		7.953430E-03
5	-2.891991E+02	0.	0.	4.602746E+01	0.		2.396782E-03
6	-3.346440E+02	0.	0.	5.326024E+01	0.		2.071297E-03
7	-9.985279E+02	0.	0.	1.589207E+02	0.		6.941691E-04
8	1.304951E+16	0.	0.	2.076894E+15	0.		-5.311672E-17
9	-4.384582E+16	0.	0.	6.978279E+15	0.		1.580874E-17
10	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
11	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
12	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
13	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
14	-1.000000E+38	0.	0.	1.591549E+37	0.		6.931472E-39
15	-1.518183E+01	4.477549E+01	7.126320E+00	7.524810E+00	1.744519E-02	3.211061E-01	4.565638E-02
16	-1.372599E+01	2.436883E+02	3.878419E+01	3.884567E+01	9.494348E-02	5.623690E-02	5.049887E-02
17	-2.413766E+02	3.315639E+02	5.277004E+01	6.527241E+01	1.291808E-01	5.885532E-01	2.871642E-03
18	-4.433366E+02	4.294579E+02	6.835035E+01	9.823631E+01	1.673213E-01	7.182600E-01	1.563478E-03
19	-1.904640E+01	6.258509E+02	9.960726E+01	9.965338E+01	2.438380E-01	3.041872E-02	3.639256E-02
20	-8.861198E+00	7.931390E+02	1.262320E+02	1.262394E+02	3.090152E-01	1.117162E-02	7.822274E-02
21	-9.682847E+01	1.029206E+03	1.638032E+02	1.645266E+02	4.009894E-01	9.366712E-02	7.158506E-03
22	-2.421634E+01	1.562336E+03	2.486535E+02	2.486834E+02	6.087025E-01	1.549694E-02	2.862548E-02

MINIMUM BLANK COMMON LENGTH REQUIRED = 7646.
BASED ON INPUT DATA AND ANALYSES REQUESTED.

SYMBOLS

In this modification to the original program PASES, the general notation has been retained. The nomenclature used in the basic development (Ref. 1) has been reproduced in Appendix A. The new symbols introduced in the present modification are defined here. The reader is referred to Appendix A for the definitions of symbols not appearing below:

A	Coefficient in spline fit, Eq. (57); also see Appendix A
a_0, a_1	Coefficients in spline fit, Eqs. (67) and (68)
B	Coefficient in spline fit, Eq. (57); also see Appendix A
b_r	Reference semichord
C	Element of discrete damping matrix; coefficient in spline fit, Eq. (57)
$\mathcal{C}$	Element of generalized damping matrix
C_h	Element of oscillatory AIC matrix
C_{hDh}	Element of damping AIC matrix
C_{hI}	Imaginary part of C_h
C_{hR}	Real part of C_h
C_{hs}	Element of static AIC matrix
$\bar{c}$	Reference chord
D	Coefficient in spline fit, Eqs. (57) and (60)
E	Modulus of elasticity
f	Cyclic frequency
g	Aeroelastic damping coefficient
I	Element of unit matrix; spline bending moment of inertia
I_{kj}	Element of spline interpolation matrix, Eqs. (74) and (75)
K_{ji}, K_{kj}	Spline parameters, Eqs. (70) and (73)
$\mathcal{K}$	Element of generalized stiffness matrix

M	Element of generalized mass matrix
P	Spline reaction force
Q	Spline loading parameter, Q=P/12EI
q	Dynamic pressure
S	Reference area
s	Reference span; Laplace transform variable
$T_{1/2}$	Time to half amplitude (negative value denotes time to double amplitude)
U	Element of matrix in canonical form for eigenvector analysis, Eqs. (45) and (47)
w	Spline loading, Eq. (56a)
x	Coordinate along spline; also see Appendix A
y	Spline deflection; also see Appendix A
Λ	Eigenvalue (=0) in eigenvector analysis, Eqs. (45) and (46)
μ	Decay rate coefficient
ρ	Atmospheric density
$(\tilde{\ })$	Denotes complex amplitude; denotes inclusion of aerodynamic damping and stiffness in $\tilde{C}$ and $\tilde{K}$, respectively.

Subscripts:

r	Oscillatory mode number in frequency lining-up process, Eq. (53)
s	Number of mode being lined-up, Eqs. (53) and (55)

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